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ABSTRACT

This project developed materials for an innovative new approach to calculus for students in business, economics, liberal arts, management, and the social sciences. With the focus on rates and accumulation of change and their interpretations in real life situations, the materials are data driven, technology based, and feature a unique modeling approach. Emphasizing fundamental calculus concepts and their relevance and utility in non-scientific careers, these materials stress the development of conceptual understanding without the traditional accent on algebraic skill and technique, emphasize group work on team projects that involve mathematical decision making and interpretation of results, and use technology as a tool for the learning of mathematics in an active, constructive environment. These materials are being implemented in a variety of postsecondary and some secondary institutions. This document also includes an executive summary containing a project overview, statement of purpose, background and origins, project description, evaluation/project results, and conclusions. Project presentations, publications, and a detailed external evaluation are included in the appendices. (Author/ASK)

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ED 417 936

Revitalization of Nonstandard Calculus

Grantee Organization:

Clemson University
Department of Mathematical Sciences
Clemson, South Carolina 29634-1907

Grant Number:

P116B20056

Project Dates:

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Ending Date: September 8, 1995
Number of Months: 36

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Grant Award:

Year 1	$ 47,273
Year 2	36,588
Year 3	33,724
Total	$117,585

SE061188

Summary

The project developed materials for an innovative, new approach to calculus for students in business, economics, liberal arts, management, and the social sciences. With the themes of rates of change and accumulation of change and their interpretations in real-life situations, the materials are data driven, technology based, and feature a unique modeling approach. With a focus on fundamental calculus concepts and their relevance and utility in non-scientific careers, the materials stress the development of conceptual understanding without the traditional accent on algebraic skill and technique, emphasize group work on team projects that involve mathematical decision making and the interpretation of results, and use technology as a tool for the learning of mathematics in an active, constructive environment. The products are being implemented in a variety of post-secondary and some secondary institutions.

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Products of the Project:

Calculus Concepts: An Informal Approach to the Mathematics of Change, Preliminary Edition, D. LaTorre, J. Kenelly, I. Fetta, C. Harris, L. Carpenter, D. C. Heath and Company, August 1995, ISBN: 0-669-39865-9

Calculus Concepts: An Informal Approach to the Mathematics of Change, Preliminary Edition Multivariable Chapters, D. LaTorre, J. Kenelly, I. Fetta, C. Harris, L. Carpenter, D. C. Heath and Company, January 1996, ISBN: 0-669-41803-4

Instructor's Guide with Complete Answer Key to accompany Calculus Concepts: An Informal Approach to the Mathematics of Change, Preliminary Edition, L. Carpenter, D. C. Heath and Company, August 1995, ISBN: 0-669-39868-3

Graphing Calculator Instruction Guide to accompany Calculus Concepts: An Informal Approach to the Mathematics of Change, Preliminary Edition, I. Fetta, D. C. Heath and Company, August 1995, ISBN: 0-669-39869-1

Executive Summary

Project Title: Revitalization of Nonstandard Calculus

Grantee Organization:
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Project Overview

The goal of the project was to develop, test, and implement innovative, up-to-date courses in Clemson University's two-semester applied calculus sequence for students in business, economics, agriculture, liberal arts, management, and the social sciences. Our objective was to design each of the courses to improve conceptual understanding and learning by involving students with new materials in a way that is significantly different from current practice: incorporating concepts and topics from calculus that are relevant and useful in non-scientific careers, providing experience with mathematical modeling, focusing on concepts and their relationships, requiring the use of technology, and conducting the courses in an active, constructive environment.

Purpose

For all their notable successes, the changes in the teaching and learning of calculus spawned by the "calculus reform" movement are seen almost entirely within the "mainstream" calculus taken primarily by those who would be scientists or engineers. Conspicuously missing have been similar reform efforts directed toward the revitalization of calculus courses designed for students in business, economics, liberal arts, and the social sciences -- courses referred to as "business" or "applied" calculus by the mathematics teaching community. While faculty in business, economics, management, and the social sciences have responded to today's contemporary need for quantitative understanding in a global economy by adding mathematics requirements which typically include one or two courses in nonstandard (business) calculus, these courses are taught almost exclusively within mathematics departments. These courses universally suffer from inappropriate content, fail to incorporate realistic applications, and are largely untouched by the graphics calculators and microcomputers that are energizing the mainstream courses. Thus, this project was conceived in response to a need to restructure the content, spirit, and methodology of Clemson's two-semester business calculus sequence so as to better prepare non-technical students for their ultimate careers.

Background and Origins

The project began in the 1992-93 academic year with the hope of improving learning and understanding of calculus concepts by involving students with a fresh, new approach in a way that was to be a significant improvement over current methods. Beginning efforts were driven by our new philosophy that students can learn the fundamentals of calculus and how they apply to real-life situations without a high level of algebraic skill. This philosophy resulted in a new goal; namely, to develop conceptual understanding rather than algebraic prowess. We felt this goal could be achieved with a new vehicle for learning: graphics calculators. Introduction of the calculators into the course facilitated a new approach -- that of examining real-world situations from graphical, numerical, and analytic perspectives. Calculators were loaned to students for the duration of their courses during the first two years of the project since it was considered inappropriate to require them to purchase their own units during periods of experimentation and development. Because of the unusual scope of the project, we were successful in obtaining substantial external support from Sharp Electronics Corporation and Texas Instruments through a loan of 140 Sharp EL-9300 graphics calculators in the fall of 1992 and 150 TI-82 graphics calculators in the fall of 1993. The calculators were loaned to students under signed loan agreements.

Project Description

The project began in the 1992-93 academic year with the development and offering of experimental prototypes of each of the two targeted courses, Business Calculus I and II. In the summer of 1993, immediately following the prototype courses, the staff modified and redeveloped each of the courses in light of the prototype experiences and the preliminary evaluations. They also resolved several significant technology-related issues that surfaced during the prototype courses. The 1993-94 academic year was spent developing and teaching two pilot versions of each of the earlier prototypes of the two courses. The instructional staff for the pilot classes was expanded to include teachers other than project participants in an effort to gather evidence of our approach in a variety of settings with a variety of instructors. The summer of 1994 was directed towards refining the pilot courses, preparing organized, written versions of the preliminary materials for both courses, and training Clemson University mathematics instructors. During the 1994-95 academic year, the newly developed courses were incorporated into the mathematics program at Clemson University by offering these courses in all sections of Business Calculus I and II. Xeroxed copies of the course materials were purchased by students at minimum cost from a local copy shop and replaced a standard textbook in both courses. The spring of 1995 was devoted to final organization of the multivariable chapters for Business Calculus II, preparing written course supplements, and organizing the preliminary version of a text containing the project materials for publication.

Project Results

The project has been immensely successful in achieving large-scale implementation of its philosophy and products on the Clemson campus. It has evolved from six prototype courses the first year to full implementation in all forty-four classes at Clemson in year three (over 1,500 student enrollments).

Because of its new focus and data-driven approach, this project is attracting national interest. During the third year of the project, thirteen institutions across the country served as informal field test sites for our materials and approach. In July of 1995, several representatives from institutions across the U. S. and Canada met in Clemson, S. C. for a three-day conference introducing the new materials. The conference focused on how the instructors could use the materials to their best advantage. The project staff gave seminars on how they used the materials in their classrooms and where they had difficulties and successes along the way. It is estimated that more than fifty institutions are using our materials during the 1995-96 academic year.

In order to document and evaluate the use of the Clemson project at selected test sites, we are fortunate to have been awarded another FIPSE grant, *Disseminating a New Approach to Calculus for Non-Technical Students* (September, 1995). Areas of evaluation at each of ten test sites include student performance, student attitudes and beliefs, impact on instructional policy and curriculum, instructor perceptions, and transportability of the project to other schools.

The popularity of the Clemson materials played a major role in the inclusion of the logistic regression model in the Hewlett-Packard HP-38 (April 1995) and the logistic and sinusoidal regression models in the Texas Instruments TI-83 (January 1996) graphing calculators.

The impact and results of this FIPSE project are described in the publication, or pending publication, of 10 articles in an array of professional mathematical journals, conference proceedings, or other professional publications and 135 presentations at national, regional and local meetings, conferences, seminars, and colloquia concerning the project.

Summary and Conclusions

The project has more than met the specific goals outlined in the original proposal. We have seen some remarkable changes over the three years: students are experiencing mathematics in a different way than they have before, with a different emphasis that genuinely interests and involves them. Although we haven't seen an improved withdrawal rate, we have seen a lower failure rate, and more importantly, much improved student attitudes. We are providing students with skills and knowledge that will likely serve them well in the future and providing them with a realization that common sense and the ability to analyze the reasonableness of answers must be part of doing mathematics.

Even though the extent to which the project is meeting its stated overall goal of improving student learning of basic calculus concepts has been difficult to measure with traditional objective measures on a comparative basis because the project represents a new paradigm for the learning of calculus, we have evidence from common final exam questions that students are retaining key concepts. The judgment of the project's external evaluator is that the performance on these questions as a whole is acceptable. Do students at other colleges who have studied this material do as well as Clemson students? The evaluation states "Clearly, if the performance is satisfactory for Clemson students (and we believe it is), then it is also satisfactory for students at other colleges -- since the profiles of the graphs [of exam question results] are basically indistinguishable."

The products of this project --the text *Calculus Concepts: An Informal Approach to the Mathematics of Change, Preliminary Edition* and its associated supplements, are being regarded by many business and mathematics instructors as benchmarks for a revitalized approach to the teaching and learning of calculus for students in non-technical careers.

Revitalization of Nonstandard Calculus
Final Report

Project Overview

The calculus courses for students majoring in business, economics, management, liberal arts, social sciences, and other non-technical majors found on most college and university campuses universally suffer from inappropriate content, fail to incorporate realistic applications, and are largely untouched by technology. This project was conceived to respond to this situation by developing, testing, and implementing innovative, up-to-date courses in nonstandard, or "business" calculus, the calculus taken by many of our future national leaders.

The primary goal of the project is to restructure the content, spirit, and methodology in Clemson University's two-semester business calculus sequence in order to improve student learning and to better prepare non-technical students for their ultimate careers. Each of the two courses involve students with fresh, new material in ways that are a significant improvement upon current practice. We seek to develop conceptual understanding of important calculus concepts with instruction and applications that are relevant and useful in non-technical careers, provide realistic experiences with mathematical modeling, focus on concepts and their relationships instead of overemphasis on algebraic manipulations and skills, require the use of technology, and conduct the course in an active, constructive environment from multiple perspectives.

By almost any account, we feel that the project has made a noticeable impact on the Clemson campus and overall has had extraordinary success. Namely,

- At Clemson, the project has evolved from six prototype classes in the first year to thirteen pilot classes in the second year, and is currently in year three in full implementation in all forty-four classes (over 1,500 students). The instruction staff has expanded from the project participants to include 14 graduate teaching assistants, 3 lecturers, and 5 other professors. All classes in year three are taught in the standard class (size 35-40) format.
- National interest in the project appears to be keen as evidenced by attendance at the mini-courses, workshops, and presentations the project staff has conducted at local, regional, and national conferences.
- Thirteen institutions across the country served as informal field test sites for the project materials and approach during the third year.

- A two-day workshop for 30 faculty from across the U. S. and Canada was held at Clemson in July, 1995. The participants interacted with the principals in this project to learn about our new content, philosophy, methodology, and technology.

- The project materials were published by D. C. Heath and Company in July, 1995 and were considered to be a major acquisition by Houghton-Mifflin Publishers when they purchased D. C. Heath in December, 1995. D. C. Heath also made a major financial commitment, other than standard marketing techniques, to the dissemination of the project materials.

- An internet discussion group was established in September, 1995 and provides all test sites with the opportunity to converse with the authors of the text and each other on topics relating to the courses and materials.

- It is estimated that over 50 colleges, universities, or secondary schools are using the text in the 1995-96 academic year.

- The publication, or pending publication, of 10 articles in an array of professional mathematical journals, conference proceedings, and other professional news media describe the impact and results of this FIPSE project.

- 135 presentations at national, regional, and local meetings, conferences, seminars, and colloquia concerning this project in whole or part have been made. Most of these were by invitation, as opposed to routine submission by contributors.

- The popularity of our modeling approach is causing changes on a national scale in current technology as evidenced by the inclusion of the logistics model in the Hewlett-Packard 38-G graphing calculator and the logistics and sinusoidal regression models in the recently introduced Texas Instruments TI-83 graphing calculator.

- The staff has been invited to submit the Clemson Project in the First Annual Awards Competition for Innovative Programs Using Technology in Mathematics Service Courses. Awards are sponsored by the Annenberg/CPB Project and NSF through a grant to Central Michigan University.

- The Clemson Project was selected for inclusion along with seventeen of the other (mainstream) Calculus Reform projects that have been written in recent years in a publication for Calculus Reform Workshop participants under Don Small's and Wayne Roberts' NSF project.

- Clemson University was recently awarded by FIPSE a grant to conduct a comprehensive, national dissemination program for the project materials. The 1995-96 academic year's portion of the grant will assist a group of ten institutions in implementing the new approach on their own campuses and document student reaction and performance, instruction of the project materials, instructor perceptions, and transportability.

Future plans include more interdisciplinary communication with and evaluation by departments whose students are using the project materials, national dissemination workshops for interested faculty, publishing a semi-annual newsletter, and preparation of the first-edition publication of the materials based on revisions responding to the project staff's classroom experiences and to test-site faculty and student comments.

Purpose

The past decade has seen many notable efforts directed towards the reform of the teaching and learning of "mainstream" calculus; i.e., the calculus taken primarily by those who would be scientists or engineers. Conspicuously missing have been similar reform efforts directed toward the revitalization of calculus coursed designed for students in business, economics, liberal arts, management, and the social sciences. These calculus courses, often referred to as "brief", "applied", or "business" calculus, are offered primarily as service courses within mathematics departments and are often taught by part-time instructors or graduate teaching assistants. Frequently, a statistics course is taken by the students populating the business calculus course(s), but there does not appear to be a general standard as to whether or not the statistics course is taken preliminary to or subsequent to the business calculus course. While faculty in business, economics, management, and the social sciences have responded to today's contemporary need for quantitative understanding in a global economy by adding mathematics requirements that typically include a course in nonstandard (or business) calculus, the mathematicians writing the textbooks for these courses have failed to produce anything but a watered-down skeletal course in "traditional" calculus that poorly serves these students' long term needs.

The business calculus courses found on many campuses often reflect content that is, at best, of marginal relevance to the long-term career objectives of the students in the course and are largely untouched by the technology that is helping to energize the mainstream calculus courses. The students in these courses often suffer from mathematical anxiety, have poor retention of algebraic skills, and see little relevance of the course material to their major fields of study or their lives. There is also ample evidence that the lecture approach very often used in these courses does not work; it fails to captivate student interest at this level and to encourage students to become active participants to their learning. Many of these students will go on to become leaders in our country's businesses, industry, and government. Since calculus is often the last mathematics course these students take, it seems imperative that their grounding in the subject should be reflective of its uses in our modern, information-based society.

In response to the apparent need for a "better way", the project sought to design, test, and implement new, relevant courses in Clemson's two-semester business calculus sequence. The project design called for developing prototype courses and refining them based on classroom experience the first year, offering pilot sections of each course the second year with further refinement during the summer, and large scale introduction into the Clemson program during the third year.

Our understanding of the initial problem that this project addresses has not changed. However, implementation of the materials made us realize that students and faculty are sometimes hampered with using the course materials because of preconceived notions of what "calculus" should be and that the format of prior courses taken by students in business calculus is partially instrumental in their initial acceptance of our approach.

Background and Origin

With the successful completion of a prior FIPSE grant to integrate graphing calculators on a broad basis in Clemson's undergraduate mathematics courses taken by students with science and engineering majors, Don LaTorre, John Kenelly, and Iris Fetta realized the need for incorporating technology in service courses taken by students with non-technical majors. After Ms. Fetta taught a trial section of business calculus in the Spring of 1992 with standard materials and graphing calculators, it became apparent that much more than just incorporating technology with a traditional text was needed for students in the business calculus courses.

Due to her innovative teaching methods and excellent teaching reputation, Cindy Harris was asked to be a project participant. The project actually began in the Fall term of 1992 with FIPSE funding and the development of preliminary first course materials. Laurel Carpenter, a doctoral student at Clemson, became very interested in the project and wrote many of the problems used in the materials while teaching a pilot section of the course. Due to the excellent and extensive nature of her work, she was invited to join the author team in the second year of the project.

One or both of the applied calculus courses offered at Clemson University are taken by students majoring in such fields as animal science, general liberal arts, agriculture, horticulture, financial management, English, visual arts, health science, packaging science, political science, marketing, philosophy, economics, history, building science, industrial management, pre-physical therapy, speech, geology, architecture BA, accounting, and a variety of other "non-technical" majors. The majority of those students enrolling in Clemson's first course, hereafter referred to as Business Calculus I, are in the College of Commerce and Industry, and the second course, referred to as Business Calculus II, is populated by students primarily in that College. We therefore decided the emphasis on applications in our materials should be divided equally between business related topics and other types of applications that would be relevant to a variety of major fields.

Calculators would be loaned to students for the duration of their courses during the prototype and pilot phases since it was considered inappropriate to ask students to purchase calculators during the development process. During the first two years, students in the second semester of the course who had not had the first course with the project materials were encouraged to change to traditional sections. Because of the unusual scope of the project and Clemson's past reputation for innovative teaching with graphing calculators, we were successful in obtaining substantial external support from Sharp Electronics Corporation and Texas Instruments. Sharp loaned us 140 EL-9300 graphics calculators in the fall of 1992. When Texas Instruments introduced the TI-82 model, it was evident to the project staff that this calculator's rich array of features and built-in models would make it easier to use with course topics. We were fortunate to obtain the loan of 150 TI-82 graphics calculators in the fall of 1993. Second semester business calculus students continued with the Sharp calculator they had used in the first course, but all first semester students began using the TI-82. All calculators were loaned to students under signed loan agreements. In year three, students are required to purchase their own calculators. Even though instructors use the TI-82, the graphing calculator manual that now accompanies the text allows students to easily use a TI-82, TI-85, or Hewlett-Packard 48G/GX calculator.

Since the majority of students in Clemson's business calculus sequence are in the College of Commerce and Industry, the project staff made a presentation to that College's Curriculum Committee in the second year of the project. We were interested in their reactions to our work and an informal assessment of the extent to which the faculty in Commerce and Industry would support our efforts. The presentation was very well received, and in the ensuing discussion the Curriculum Committee confirmed their strong support for the changes we are making.

Project Description

When the ideas for this project began to emerge, staff participants discussed the job requirements for students in non-technical mathematics courses with several regional companies and listened to many "math in business and industry" related talks at regional and national mathematics meetings. Among the many issues discussed, the following job requirements were mentioned in some form by everyone interviewed:

- good communicator (writing skills)
- presentation skills (verbal skills)
- logical thinker
- technology skills
- group problem solving (essential)

We examined many different business calculus texts and could find none that addressed more than one of these requirements. It was also evident that the high level of algebraic manipulation required by most traditional texts overwhelms these non-technical students who are not strong in algebraic skills and stifles their progress in calculus courses.

The project staff felt that these students needed a fresh, new approach to calculus concepts relevant to their careers that are not highly technical. We were convinced that the development of conceptual understandings, not the mastery of algebraic technique, should be the guiding philosophy of a modern business calculus course and decided to focus on key calculus concepts and their relevance to a world of change. We agreed to carefully trim away excess topics that tend to overwhelm and stifle the mathematical development of non-technical students. To these ends, we decided the materials should feature a new classroom dynamics with less lecturing, more student activity, interpretation and mathematical decision making, and group work on team projects. Our objectives included incorporating concepts and topics that are relevant and useful in non-scientific careers, focusing on concepts and relationships, and requiring the use of technology so that the course could be conducted from graphical and numerical as well as analytical perspectives.

To accomplish our goal, the project staff had to reconsider our understanding of important ideas associated with the learning of mathematics. Several of these ideas are unmistakably clear, and they are foundational to many projects that involve reform, including ours. We use a constructive approach in that we feel students learn best by not simply remembering what they were told in a lecture but by privately constructing their knowledge by their experiences with mathematics. A new classroom environment is strongly suggested with our materials because learning takes place best when students are immersed in an environment where inquiry is a natural occurrence. We feel that ongoing dialogues between students and teachers, and between students in groups, are crucial to the development of genuine understanding. The use of technology is essential with our materials for connections to the world with real data, the break down of barriers imposed by the traditional emphasis on purely analytical treatments, and the opportunity for insight through graphical and numerical representations of problem situations.

We also investigated the major areas of applications for math in industry. These appear to be:

- research (developing processes/product -- limited data)
- engineering (designing future products -- no data)
- manufacturing (process engineering/maintenance -- lots of data)
- marketing (selling the product -- lots of data)

Since business students are most likely to be involved in manufacturing and marketing, another objective we considered essential was to provide modeling experiences using real data. In fact, the use of technology allowed modeling to be a central theme of the project materials. Introduced at the onset are linear, quadratic, cubic, exponential and logistic models. We have found this approach to be extremely effective in capturing student interest since it is something they have not encountered in their past experiences and it presents an opportunity to analyze real-world data. Most students have commented that it is this feature of the materials that they feel shows the relevance of the mathematics in the course to their lives and adds validity to the topics under study. The project staff has tried to ensure that the subtleties of modeling are not the issue; rather, elementary models are used to obtain functional relationships between variables. The functions represented by the models are the ones on which the students conduct their calculus investigations.

Other ideas associated with the learning of mathematics that have not been as evident from the reform of mainstream calculus, but which have emerged as the true hallmarks of our work in calculus for non-technical students are:

- **a new, narrower focus**

  The overarching theme of our material is rates of change and their interpretation in non-technical settings: the derivative as a rate of change and the integral as the accumulation of change.

- **mathematical models as a vehicle for learning**

  We discuss linear models (constant rates of change), exponential models (constant percentage change), quadratic models (constant force for change), cubic models (smooth, transitional change), and logistic models (exponential change with limiting conditions). This presents opportunities to analyze real data that adds validity to the topics under study and to put to rest the classic question, "Where is this ever going to be used?" Interpretation and discussion of results from calculus investigations on the functions given by the models becomes meaningful in the context of real life situations.

- **an interplay between the discrete and the continuous**

  Many everyday, real-live situations involving change are discrete processes. Such situations can often be represented by continuous mathematical models so that the concepts, methods, and techniques of calculus can be brought to bear.

- **a new role for algebra**

  When everyday, real-life situations that are discrete in nature are represented by continuous mathematical models, the role of algebra is to describe and to enable us to reason with the quantities that are undergoing change. This is in contrast to the traditional role of algebra as a collection of organized manipulations and procedures that are applied to obtain a numerical answer to a well-formulated problem.

- **mathematical interpretation and decision making**

  Of equal importance to understanding concepts of calculus in the context of change is the ability to correctly interpret the mathematics in real-life situations. The ability to "make sense" of mathematics is vital to believing in its value and appreciating its usefulness in our lives.

Implementation of this new approach and materials began with 6 prototype courses in the 1992-93 academic year. Due to the sequential nature of the courses, the major effort was directed towards Business Calculus I with 2 prototype courses taught during the Fall term, 2 in the Spring term, and one in the Summer session. One very tentative prototype of Business Calculus II was taught that Spring term. All prototype classes were randomly selected from regularly-scheduled courses. These prototype courses accomplished several goals: a clearer understanding of the students in the courses, a better understanding of the topics we should and should not include in the course, and an affirmation that the changes we were proposing can work in practice. During the 1993-94 school year, we refined and added substantially to the prototype materials to develop a comprehensive manuscript for single-variable calculus. That year we offered 4 pilot sections of Business Calculus I in the Fall and 3 in the Spring. Two pilot sections of Business Calculus II were taught in each of the Fall and Spring terms.

The summer of 1994 was spent further refining the comprehensive manuscript and writing the multivariable calculus chapters for Business Calculus II. In April, 1994, the project staff, after lengthy discussions with the entire mathematical sciences faculty, received approval to use the project materials in all 44 sections of Business Calculus I and II offered during the 1994-95 academic year. Over the course of the project, faculty and graduate teaching assistant training has been an ongoing process.

Project Results

The results of the project will be summarized in four areas: Impact on Students, Impact on Faculty, External Impact, and External Evaluator's Report.

Impact on Students

Unlike students in more technical fields who more clearly understand the need to develop genuine understanding of mathematics concepts, the typical business calculus student views courses in mathematics as requirements to get out of the way as quickly and as painlessly as possible. We initially knew of this problem and feel that we now genuinely interest these students by involving real data. It is also becoming evident that students who have had prior experience with "reform" previous courses, especially those using technology, adapt much faster to our new philosophy. Students who are accustomed to a lecture format wherein formulas and procedures are to be memorized and routinely applied to template problems are often initially uncomfortable with our attempts to "involve" them in thinking about the concepts and relating them to situations in new and non-traditional ways. However, attitude surveys administered at the end of each semester to students in all sections using the project materials clearly indicate that our students are overwhelmingly positive about the changes we are making. An evaluation team has been gathering information on student learning and reaction since the onset of the project. Those results are summarized in the external evaluator's report.

One problem area arose that we did not anticipate. Students in these courses have traditionally relied on help outside the classroom and the instructor's office. They hire mathematics graduate students as tutors, athletes in the courses rely on tutoring provided by the athletic department, and many students ask for assistance from friends who have taken a traditional version of the course. It soon became obvious that "traditional" tutoring by persons not having knowledge of the calculator or the new approach was only confusing students. We therefore found it necessary to inform all outside tutors of the changes we were making and offer them training.

We have seen some remarkable changes at Clemson. Many students are experiencing mathematics in a different way than they have before, with a different emphasis that genuinely interests and involves them. Although we haven't seen an improved withdrawal rate, we have seen a lower failure rate and, more importantly, much improved student attitudes. Student journals have been used as an effective means of communication by student about their concerns. Beginning with the Fall 1994 term, "jump-start" sessions offered at night during the first week of class have been effectively used by students without graphing calculator experience. Also since the Fall 1994 term, Clemson has provided student support for four hours a week an experienced graduate student available for tutoring help with Business Calculus I materials and calculator use.

Impact on Faculty

Due to the nature of the project materials, it was obvious from the start that communication of the philosophy of the project and training in using the materials for faculty as well as graduate teaching assistants was a necessity. This was accomplished by summer workshops and weekly seminars during the Fall and Spring terms.

We also have become aware that we are causing faculty at Clemson and students who have previously been exposed to calculus topics to redefine their concept of "calculus". While we initially thought there may be some resistance to using technology, this has not been a cause of concern by faculty or most students. However, group work and projects, less lecturing, writing explanations in English rather than using only mathematical symbols, interpreting answers, modeling, and the lack of extensive algebraic manipulation are new concepts that must be examined and determined to have validity in the courses by those with traditional backgrounds.

During the first two years, other Clemson faculty became aware of changes in the business calculus courses since they were asked to grade presentations of course projects. One was a poster project and the others were oral presentations. Many faculty now perceive that we are providing students with these skills and knowledge that will likely serve them well in the future:

- an understanding that numerical answers are meaningless unless accompanied by a coherent statement of interpretation.
- an awareness that sometimes mathematical models can describe well the behavior of the world around us.
- an understanding that things change, and that knowing how fast they change can be useful in a variety of situations.

- a realization that mathematics is not all black and white, and that common sense and the ability to analyze the reasonableness of answers must be part of doing mathematics.

External Impact

As the project progressed, we found that many other schools also were dissatisfied with their current nonstandard calculus courses. Since we truly believe the Clemson Project offers a relevant, realistic, and much improved approach to these courses than that offered by traditional texts, dissemination of our materials became another aspect of the project. As part of the third year activities, we conducted a three-day, residential workshop on the Clemson campus during the summer of 1995 at which 30 faculty from across the U. S. and Canada interacted with the principals in the project to learn about our new content, philosophy, methodology and use of technology. We also recently established an internet discussion group for all test sites and other interested faculty using the project materials. Planning for a semi-annual newsletter for project participants is currently under way.

The project has shown evidence of strong interest by other colleges and universities across the country with thirteen informal test sites using the materials in 1994-95 and over 50 schools using the preliminary edition of the test during the 1995-96 school year. We had not anticipated secondary schools using our materials, but many have become interested. We estimate there are currently four high schools using the project materials.

External Evaluator's Report

This document is the external evaluation report of the FIPSE Project, 1992-1995, Revitalization of Nonstandard Calculus, at Clemson University under the direction of Donald LaTorre. The report and associated Appendices contain the real names of students and instructors involved in the project and should not be reproduced in any form without their expressed consent. *Dr. James W. Wilson, University of Georgia*

A. Project Overview and Chronology of the Evaluation Process

The project, *Revitalization of Nonstandard Calculus,* was funded from the U. S. Department of Education's FIPSE program in 1992. Its agenda was to improve student learning of basic calculus concepts by developing materials and pedagogy quite different from the current practice. The aims included the following:

a. Incorporate concepts and topics from calculus that are relevant and useful in non-scientific careers.

b. Provide experience with mathematical modeling.

c. Focus on concepts and their relationships.

d. Require the use of technology.

e. Conduct the teaching of calculus for non-scientific career students in an active, constructive environment from multiple perspectives.

Each of these aims are a variance with traditional practices. With all of the discussions of reform in scientific calculus courses and reform in the K-12 curriculum, very little attention had been paid to the courses for the non-scientific career students.

Our work on the evaluation began in conjunction with Don LaTorre in the conceptualization and planning of the project. Professor Wilson had participated in previous innovative projects at Clemson and Ms. Searcy was once a graduate assistant at Clemson, teaching the Business Calculus courses offered there. We have maintained continuous contact with the project form 1992 to 1995, although distance and responsibilities prevented as much contact as we would have liked. It is important to recognize, however, that we have worked with the project staff and instructional staff at Clemson to develop a comprehensive description and documentation of the project. We have cooperated on the development of instruments, and we have worked to make use of the instructional framework (e.g., student projects, student reports, syllabi, problem sets, etc.) for sources of information.

In the summer of 1993, Ms. Searcy was a participant observer in the first teaching of the developmental course. This involved being on the Clemson campus, attending all classes, talking with students, talking with instructors and the project staff, gathering materials, recording observations, and becoming very familiar with the material as it was developed. In the fall of 1993, students presented final projects using their mathematical modeling to teams of graders. We participated in this process as observers and have continued to do so in subsequent terms.

Student opinion/feedback questionnaires have been collected with each class taught at Clemson. We have collected sample student projects, student journals, and tests. During the 1994-95 school year, 13 institutions volunteered to test the new materials. We were able to interview representatives from some of these schools. Also, in 1995, a small set of test items were proposed by the project staff for gaining additional student performance data. These were used in the evaluation at Clemson and some of the volunteer test sites. Much of the material on which we are building the evaluation report is documented in the attached evaluation appendices.

B. The Materials

As a graduate assistant in the Mathematics Department at Clemson University (1989-1992), Ms. Searcy taught both Business Calculus I and Business Calculus II. This was just prior to the revitalization effort currently underway at Clemson. Although it is hard to capture what happens in a classroom in two or three sentences, her classes were basically characterized by a lecture format that closely followed the textbook (*Calculus for Business, Economics and the Social and Life Sciences*, Hoffman and Bradley, 4th Edition, McGraw-Hill, 1989), no technology use other than a standard scientific calculator, and some classroom discussion. Assessment was in the form of ten pop quizzes, four major tests, and a comprehensive final exam. Students were not allowed to work cooperatively on these assessment items.

In the Fall of 1992, Ms. Searcy came to the University of Georgia as a Ph.D. student in the department of Mathematics Education. Her advisor, Professor Jim Wilson, had been asked to be the primary evaluator for the Business Calculus Revitalization Project at Clemson that began that Fall. In the Spring of 1993, we met with Don and later with the rest of the project staff to discuss how the evaluation would proceed. We reviewed the activities of the prototype Calculus I course that had been offered Fall of 1992 and considered the proposed goals for the project. If their aspirations were achieved, the Business Calculus experience at Clemson would have a very different composition than the traditional one described in the above paragraph. Everything would take on a more dynamic nature.

The plans were that students would no longer be working with pre-fabricated functions; they would construct them from raw data using a modeling process. In fact, the curriculum would be a reflection of the project staff's "vision", resulting in a series of course materials written by the staff. Graphing calculators would provide greater access to a variety of representations of everyday phenomena. Students would work cooperatively and be involved in the conceptual development of topics from both a mathematical and an application point of view. Although tests would still be important, assessment would have to reflect these changes.

The draft materials developed with these thoughts in mind, went through a multitude of revisions, and involved the efforts of several individuals at Clemson University. In January of 1995, D.C. Heath and Company was contracted to publish these materials. The first copies of the preliminary edition of *CALCULUS CONCEPTS: An Informal Approach to the Mathematics Change, Single Variable Edition* came off the presses in late July of 1995. Revisions of that edition are being made at the time of this report. The multivariable calculus chapters for these materials were published as a separate supplement in January, 1996.

C. The Operation of the Project

During the Fall of 1992, Professor LaTorre taught a Business Calculus I course and Professor Fetta taught a Business Calculus II course with hopes of building a rough outline of what the new curriculum should look like. Spring semester of 1993 was extremely busy for the project staff as they organized, edited, and embellished on these thoughts and materials. Of course, like most cooperative efforts, the collective "vision" meant collaboration, communication, and compromise that resulted in drafts, revisions and re-revisions. This was not a trivial process. The goal was to have the first trial run-through of the materials during the second summer session of 1993 that ran from June 30th to August 4th. Observations from that class can be found in Appendix III-A. As the June 30th date approached, only the first part of the written materials was ready for distribution. It was agreed that the staff would continue to work on the remaining parts while John Kenelly began teaching the experimental Business Calculus I course. Subsequent parts would be printed and distributed to students as they were needed in class.

Fall semester 1993 was spent refining the Business Calculus I materials and getting the bulk work done on the Business Calculus II materials. In the meantime, the new materials were being used in three Business Calculus I courses. This trend continued for the remainder of the 1993-94 academic year. In the fall of 1994, a massive change was instituted at Clemson University. All Business Calculus I courses were taught with the revised materials. This brought a challenge to the project staff to prepare graduate students for handling many sections of this course. Professor Harris and Professor LaTorre held seminars before the school year began to introduce the materials to the new instructors and then instituted weekly instructor meetings throughout the remainder of the fall term. This process was repeated in the Spring of 1995 when the Business Calculus II course went 100% with the new multivariable materials.

As mentioned before, several institutions that had heard about the project from various sources, volunteered to try the materials during the 1994-95 school year. This gave way to a formation of a new project idea concerning the dissemination of the new materials to other institutions. In July of 1995, several representatives from institutions across the U. S. and Canada met in Clemson S. C. for a three-day conference introducing the new materials. A list of those institutions, at that time tentatively scheduled to use the Calculus Concepts materials, is found in Appendix III-B. The conference focused on how the instructors could use the materials to their best advantage. The project staff gave seminars on how they used the materials in their classrooms and where they had difficulties and successes along the way.

D. Student Performance

1. Student Projects

There are many elements to student performance. In this environment, student projects were a new direction for student performance. They were new to the students, new to the faculty, and problematic. Yet, from the student responses, from the inherent logical relevance and potential of the materials, and from the new opportunities that the use of technology provided, it was clear to the staff that this element of student performance was essential for the course being developed. It had the students engaged in problems that would not and could not be encountered in other settings.

The project staff decided to try the final project presentations again in the fall of 1993 with hopes that results would be better than what happened during the previous summer. Professor Wilson and Ms. Searcy were invited to Clemson for the two days of student presentations at the end of the Fall semester. We were both able to attend the first day, while only Ms. Searcy attended the second day.

The project chosen (Appendix III-C) for that Fall was one asking students to pretend they were heading up a fund raising effort for the Mathematics department. The students had to take polls and gather their own data which added a new twist to the project. This was a complex situation that required students to do a great deal of thinking on many issues. The presentations covered the spectrum, ranging from incredibly poor attempts where students didn't even realize that they needed a model to very nice efforts where students used ingenuity and hard work to make a good project. Ms. Searcy did not receive copies of the written projects, however, Appendix III-D contains summaries of some of the students' presentations. (The project staff video taped all final presentations that Fall.) One lesson learned from the summer was that there was a need for specific guidelines for the graders. A copy of the assessment guideline for this project is found in Appendix III.

Up to this point, little attention had been given to the Business Calculus II class. Professor Fetta was using some of the rough ideas for the new materials in this class. She was also finding different ways to assess students' knowledge. During the Fall of 1993, Professor Fetta required her Business Calculus II class to keep a journal of their experiences in her class. They were also asked to work cooperatively on projects and do presentations. Part of the students' final grades came from the compilation of a student portfolio. The requirements for this portfolio can be found in Appendix III-F. These portfolios came into our possession during the Winter of 1994. They made for some interesting reading. Students were very candid with their responses on occasion. It would be marvelous to be able to include all of these portfolios with this report; however, that would be quite tiresome.

Ms. Searcy has instead taken portions from several of these portfolios (Appendix III-G) as indicators of how students were coping with the Business Calculus II experience. Professor Fetta later told Ms. Searcy that she was unable to continue the portfolio evaluation due to the amount of time it required. However, she went on to emphasize that she still believes that it is a worthwhile idea and would someday like to re-institute it in her classroom.

Additional data from final project presentations came from the Spring 1995 semester. Ms. Searcy was able to attend several of these presentations that were held on April 24, 1995. Students were given a choice of two projects that they could present as their final project. One of the choices was an updated form of the Doubling Time project that was used in the Fall of 1993 (Appendix III-H) and the other choice (Appendix III-I) was a project dealing with the warning of drivers about upcoming toll booths on a new Super Highway (like the Autobahn) in Europe. Since the decision was made in the Fall of 1994 to have all Business Calculus classes taught use the project materials, there was a tremendous increase in the number of presentations to observe. That also meant that more people had to be involved in the assessment process of these projects. Ms. Searcy did not receive a specific assessment guideline for these projects; however, a grading sheet (Appendix III-J) was given to each person involved in the assessment process.

There was an extreme range of quality in these presentations, which was expected based on prior observations. Appendix III-K gives a summary of some the presentations (for both projects) that Ms. Searcy observed while she was at Clemson. Although the presentations were interesting in their own right, Ms. Searcy was fascinated by the different approaches used by different graders. Some had a particular question they liked to ask. One person would ask questions about all aspects of the projects, whereas another wouldn't ask many questions. One of the graders was a novice. Although he had been sitting in on many of the Business Calculus I classes, he had not taught the course. Most of the graders were Calculus I instructors who already had a tremendous amount of exposure to both projects as students worked on them. This particular grader was not afforded that luxury. That made his job very difficult, and it took him awhile to familiarize himself with the product as well as the process.

As time progresses, the ideas of projects and presentations as assessment opportunities are evolving to a more complex stage. With each trial, instructors are able to make adjustments in all aspects of the assessment process to make it more beneficial for both themselves and their students. It is evident that the instructors are learning a great deal about what their students know and don't know from these projects. Although it will be a challenge to construct more project ideas that will provide students with a chance to be creative in exhibiting their knowledge, it seems to be well worth the effort.

2. Test Data

A recurring theme is the call for performance on tests. The reality is that the old assessments on tests and quizzes concentrate primarily on procedural knowledge and are not sufficient for assessing the range of student performance desired or accomplished in *Calculus Concepts: An Informal Approach.*

Still, the demand and expectation of performance on such tests and quizzes is recognized. Instructors still give quizzes and tests; colleagues still look at the most innovative and complex project report and say, "okay, but can they differentiate functions or evaluate integrals?" The project staff recognize that developing procedural skills is a part of developing knowledge. Therefore, a small set of test items appropriate to both traditional classes and project classes were developed for instructors to use. These are in Appendix III-L.

Performance data on the items for the Business Calculus I were obtained from 273 Clemson students and from 247 students at six other colleges during Spring term. These data are tabulated in Appendix III-M. The percent of students responding correctly to each item was determined, for Clemson students and for the other six colleges pooled, and these percents are depicted on the graph on the following page. The *p*-values range from 0.40 to 0.98 and show the coverage and variability one would expect with such data.

One question to be addressed is "Is this satisfactory performance on these items?" Our judgment is that the performance as a whole is acceptable, and further, the items provide useful formative data for the project staff.

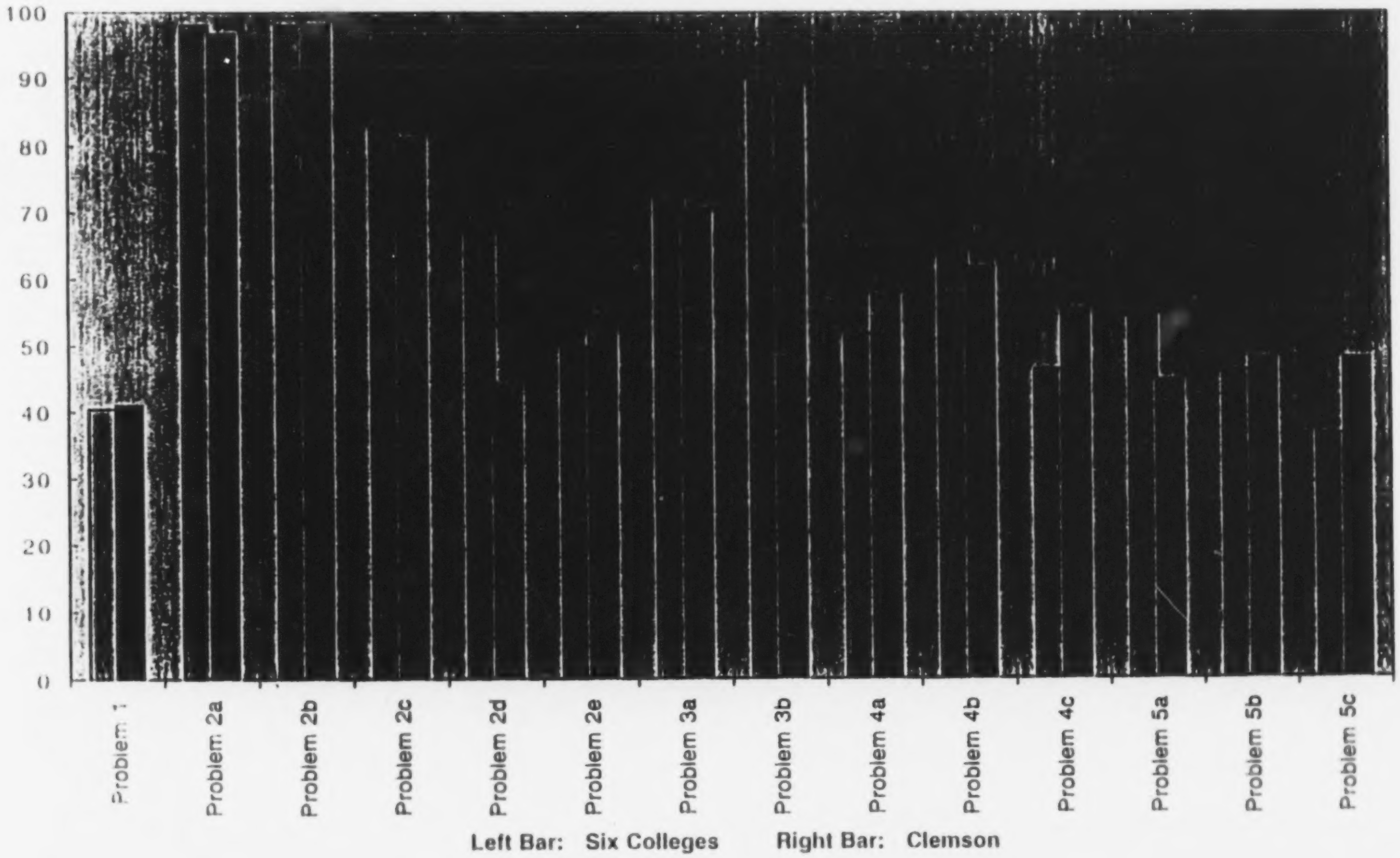
A second issue, addressed here in Section G, is whether students at other colleges who have studied this material do as well as Clemson students. Clearly, if the performance is satisfactory for Clemson students (and we believe it is) then it is also satisfactory for students at other colleges -- since the profiles of the graphs are basically indistinguishable.

E. Student Opinions

A student questionnaire was developed to provide project staff feedback from students at every step of the development process. These questions are presented and the data is summarized on two inserted pages.

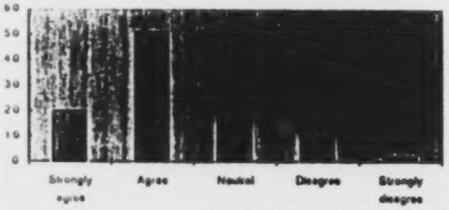
On the page following the performance data graphs, the first insert presents data from 1074 Clemson students in 48 sections taught by 20 different instructors. In general, the profiles show that the students are responding positively to the aims of the course -- reducing emphasis on algebraic manipulation, incorporating modeling, using multiple viewpoints, viewing the derivative as rate of change, and using the graphics calculator. About 85% of the students feel they are learning as much or more than students in traditional classes.

Comparision of Clemson Student Performance with Student Performance at Six Colleges
Percent Correct on Each Problem

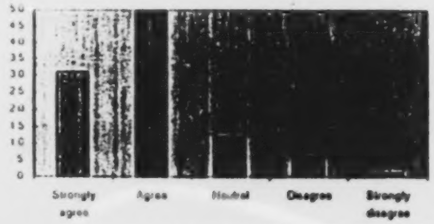


Student Opinion Survey Percents – Clemson

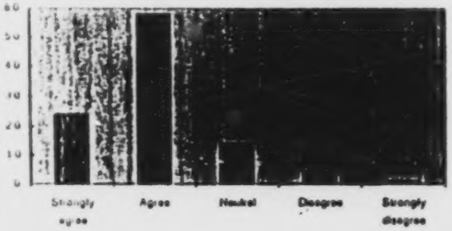
1. We have made a deliberate effort to help you learn fundamental concepts of calculus *without emphasizing algebraic manipulation and skill.* To what extent do you agree with this approach?



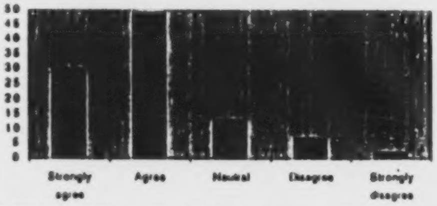
2. In this class we have *incorporate a modeling approach,* that is, given some data, construct a mathematical model that "fits" the data, to use this model to address the questions of interest. To what extent do you agree with this approach?



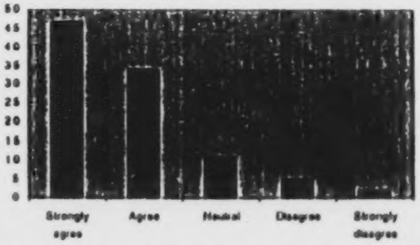
3. We have also tried to examine problems from a variety of viewpoints, e.g., *graphically, numerically, and algebraically.* To what extent do you agree with this approach?



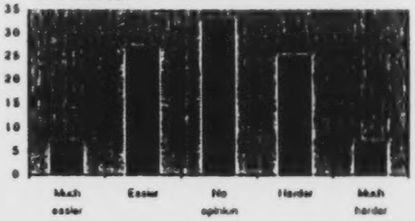
4. One way that we have changed our approach is to focus on *interpreting the derivative as a rate of change,* and the integral as the result of change, instead of a collection of rules and procedures. To what extent do you agree with this approach?



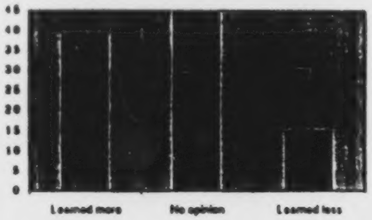
5. We believe that a *graphics calculator* can be a valuable tool in helping students to learn and to do mathematics. To what extent do you agree with this?



6. What is your assessment of the level of difficulty of this course, compared to your perceptions of the more traditional versions of this course?



7. *Are you learning more, or learning less,* than comparable students in more traditional sections of this course?



Note: This is composite data in percent making each response for students at Clemson over the three years of the project. It includes data from 1074 students in 48 sections taught by 20 different instructors

The second insert, the following page of graphs, presents the same date pooled over approximately 200 students in six different college during Spring 1995. The profiles of response are essentially the same as what has developed over the three years at Clemson.

We have also included (Appendix III-N) several student comments elicited at Clemson and some of the other test sites through the above mentioned questionnaire and instructor assigned essays.

F. Institutional Impact

We feel that perhaps the most profound element of this evaluation, one that is ignored in most such efforts, is in the *institutional impact* of this project. Essentially the FIPSE funding was to experiment with an alternative approach, and it was reasonable to expect a prototype worthy of further study or development. However, this investment has already transformed the situation at Clemson and is beginning to make impact on several schools across North America.

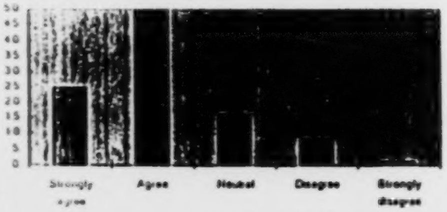
After our initial meeting with the project staff about constructing a prototype course in early 1993, we were skeptical at the changes that were being proposed in the structure and teaching of the Business Calculus courses. It seemed an immense challenge to try to implement all the planned modifications within the constraints of one project. Yet, a rather awesome metamorphosis took place during the project's three years at Clemson University. In that amazingly short period of time, the implementation has moved from a handful of pilot sections to an *institutional policy* decision to transform the entire calculus for non-scientific career students. Effective in the third year of the project, all Clemson Business Calculus courses were using the project materials. It is a remarkable mobilization of resources and influence to bring about institutional change in situations, as in mathematics departments, where academic anarchy is a cherished tradition.

Of course, there are always two sides to every coin. With the full scale implementations of the materials came an unforeseen obstacle that has beset instructors in the second course of the sequence. The population of this course seems to be about 50 percent students coming directly from the first course. The other half of the population is either transferring students from other institutions, students that have changed majors and have had one semester of engineering calculus, students with AP credit for the first calculus course, or seniors who have put off the second course until their last year. The major difficulty lies in the fact that the students that fall in these four categories have not been exposed to key elements that compose the framework of the project materials: modeling, interpretation of mathematical information, communication of mathematics in a variety of ways, and the use

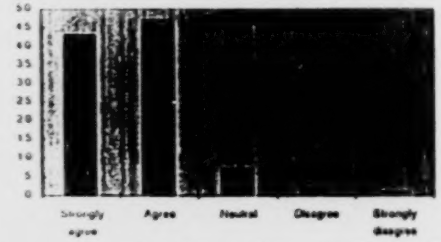
Student Opinion Survey Percents Other Schools

Spring 1995

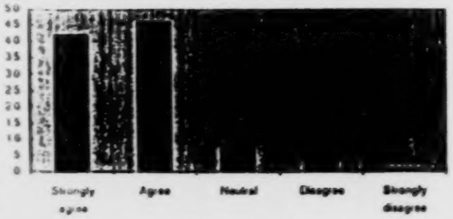
1. We have made a deliberate effort to help you learn fundamental concepts of calculus *without emphasizing algebraic manipulation and skill.* To what extent do you agree with this approach?



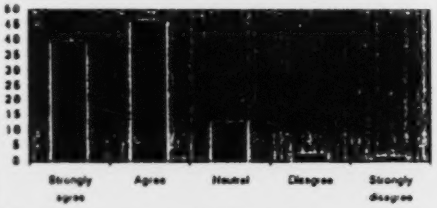
2. In this class we have *incorporate a modeling approach,* that is, given some data, construct a mathematical model that "fits" the data, to use this model to address the questions of interest. To what extent do you agree with this approach?



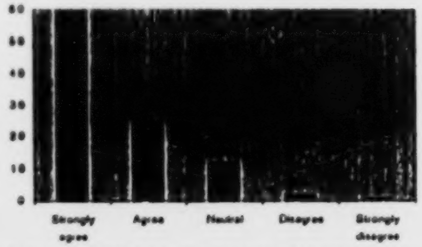
3. We have also tried to examine problems from a variety of viewpoints, e.g., *graphically, numerically, and algebraically.* To what extent do you agree with this approach?



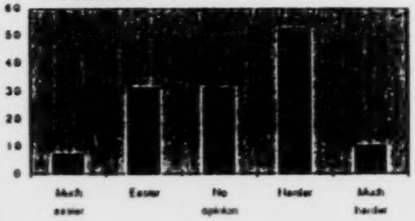
4. One way that we have changed our approach is to focus on *interpreting the derivative as a rate of change,* and the integral as the result of change, instead of a collection of rules and procedures. To what extent do you agree with this approach?



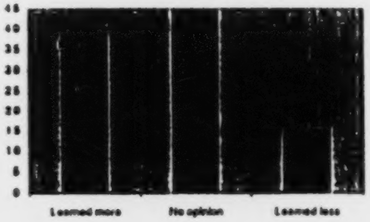
5. We believe that a *graphics calculator* can be a valuable tool in helping students to learn and to do mathematics. To what extent do you agree with this?



6. What is your assessment of the level of difficulty of this course, compared to your perceptions of the more traditional versions of this course?



7. Are you *learning more, or learning less,* than comparable students in more traditional sections of this course?



Note: This is composite data in percent making each response for students at

East Texas State University
College of Charleston
Lenoir-Rhyne College
Greenville Technical College
Kennesaw State College
Elon College

all in classes in Spring 1995. The data is from approximate 210 students in 10 classes across the 6 sites. Each of these sites was in their first term of using the material.

of technology. The staff is still trying to come up with a solution that will best benefit those students and instructors.

Another source of impact from the project has come in the form of a contract with D. C. Heath (which was taken over by Houghton-Mifflin effective January 1996) to publish the project materials. The immense effort by the staff to produce these materials in such a way as to reflect their dynamic philosophy has been rewarded. A representative of D. C. Heath feels that this is one of the largest acquisitions the company has made this decade. They are excited about the prospect of working with Clemson on disseminating these materials.

Finally, the impact of this project has been felt far beyond the walls of Martin Hall in Clemson, S. C. As mentioned before, several schools piloted the materials during the 1994-95 year. We were fortunate to be able to talk to several representatives from these schools during the conference held at Clemson in July of 1995. A synopsis of comments made by these individuals can be found in Appendix III-O. A couple of common remarks were that it took a great deal of time to prepare for teaching the first course in this sequence and that they didn't feel they had enough time to implement student projects with in the time allotted to teach the course.

It is quite evident that this project was not an invisible, watery, reform impostor standing still, but rather it was a driving force of change at Clemson University. There is still a great deal of momentum behind this project today. It will be quite interesting to see what the next three years will bring.

G. Transportability

One concern for these materials, for this approach, is that of "only at Clemson." That is, will these materials and this approach be transportable?

The project has addressed this in multiple ways. From the very beginning, there has been the feeling that the materials and pedagogy of this project was needed on a far wider scale. Thus, as the staff has gained experience with the development and implementation, they have found themselves at the center of an intense interest in what is being done. They have been thrust into a national spotlight by their work and there is extensive demand for assistance to try the materials elsewhere.

In Spring 1995 instructors at 13 colleges used the material and approach from the project -- to the extent they could implement it -- in an *informal* tryout. Comments from several of these instructors can be found in Appendix III-P. As mentioned before, we have test data, so far, from six of these sites. Given that it was the first time and the limited experience, one might expect some glitches, but there appear to have been few.

In fact, the test data in Section D-2 and the student opinion in Section E indicate in the strongest possible terms that this material is transportable. That is, the students in these six colleges responded to the material in about the same way as at Clemson.

Only one of the colleges so far (East Texas State) has been able to provide data on both a traditional and a project class. The comparison is clearly in support of the project class (Appendix III-M), but there just is not enough data for a conclusion.

Summary and Conclusions

The project has been immensely successful in achieving large-scale implementation of its results on the Clemson campus. Even though the extent to which the project is meeting its stated overall goal of improving student learning of basic calculus concepts has been difficult to measure with traditional objective measures on a comparative basis because the project represents a new paradigm for the learning of calculus, we have evidence from common final exam questions that students are retaining key concepts. The judgment of the project's external evaluator is that the performance on these questions as a whole is acceptable. For the first time, many of these students are interested and actively involved in learning mathematics. Of prime importance is the fact that most of them also see a need and use in their future careers for the mathematics they are learning.

The products of the project, the text *Calculus Concepts: An Informal Approach to the Mathematics of Change, Preliminary Edition* and its associated manuals, are being heralded as the first true reform project in nonstandard calculus. With the review process being accomplished through actual implementation in the classroom at many different institutions of higher learning, we are preparing a first edition of the text that will truly serve the needs of a diverse population of students. Finally, we are convinced that the impetus for change brought forth by this project is both strong and long-lasting. We already know of several efforts by faculty at other schools to "follow in our footsteps" with competing publishers.

Even though four of the authors of the materials will no longer be actively teaching at Clemson at the conclusion of the 1995-96 school year, all of us have plans to remain with the project and insure that it gets better as it matures.

Teaching for the first time with the project materials, as is true of any reform course, requires considerable amounts of time and effort. Advice to others using the project materials is to consider communication of prime importance. Students must be encouraged to communicate between themselves so as to gain confidence in their procedures and be able to determine the reasonableness of their answers. Professors must communicate with students to discern their reactions to the materials and the best method of implementation on their campuses, faculty must communicate with other faculty teaching the course to discuss methods of implementation and other concerns, and faculty should communicate with the authors through reading the suggestions in the Instructor's Guide and taking advantage of professional workshops and the internet discussion group.

Index of Appendices

Appendix I	Project Publications
Appendix II	Project Presentations
Appendix III	External Evaluation Appendix
Appendix IV	Information for FIPSE

APPENDIX I

Project Presentations

including presentations referencing
the project and its products

Presentations

Donald R. LaTorre

1. NSF Calculus Principal Investigators Annual Meeting, Washington, DC, October 24-25, 1992.
2. Mathematics Colloquium, Wofford College, Spartanburg, SC, October 29, 1992.
3. Fifth International Conference on Technology in Collegiate Mathematics, Chicago, IL, November 13-15, 1992.
4. South Carolina Council of Teachers of Mathematics Annual Meeting, Hilton Head, SC, November 19-20, 1992.
5. Mathematical Association of America Annual Meeting, San Antonio, TX, January 14-16, 1993.
6. New Jersey College Calculus Consortium, April 16-18, 1993.
7. Conference: Preparing for a New Calculus, Monticello, Illinois, April 22-25, 1993.
8. East Tennessee State University Mathematics Colloquium, April 27, 1993.
9. Invited participant in the 1993 National Academy of Sciences Convocation *Reinventing Schools: The Technology* is *Now*, Washington DC, May 1993.
10. San Jacinto Fourth Conference on Technology, Houston, TX, July 23-24, 1993.
11. Invited Colloquium: "Calculators and Calculus", Bob Jones University, Greenville, SC, October 14, 1993.
12. Workshop: "A Teacher's Introduction to the TI-82 Calculator: Applications in Business Calculus", South Carolina Council of Teachers in Mathematics, Greenville, SC, November 19, 1993.
13. "Restructuring Learning in Business Calculus", to the Curriculum Committee, College of Commerce and Industry, Clemson University, March 1, 1994.
14. Presentation: "Business Calculus: Modelling Change with Real Data", at the MAA Southeastern Regional Meeting, Carson-Newman College, Jefferson City, TN, April 9, 1994.
15. Invited Workshop: "Business Calculus using the TI-82", DeKalb Mathematics Conference, DeKalb College, Clarkston, GA, April 29, 1994.
16. Invited participant in the 1994 national NSF conference *Project Impact: Disseminating Innovation in Undergraduate Education*, Washington, DC, June, 1994.

17. "Business Calculus: Restructuring Teaching and Learning for a New Era", Fifth Conference on Technology, Houston, TX, July 9, 1994.

18. Minicourse: "A Teacher's Introduction to the TI-82 Calculator", Third Carolinas Mathematics Conference, Charlotte, NC, October 6, 1994.

19-20. Two presentations on the Clemson business calculus project: one to the business and mathematics faculty, the other to the MBA students and faculty, Elon College, Elon, NC, October 17, 1994.

21. Four-hour Minicourse: "Revitalizing Business Calculus", American Mathematical Association of Two-Year Colleges National Meeting, Tulsa, OK, Nov. 5, 1994.

22. Four-hour Workshop (with Iris Fetta): "Business Calculus: Modeling with the TI-82", Seventh International Conference on Technology in Collegiate Mathematics, Orlando, FL, November 17, 1994.

23. Address: "Business Calculus: Restructuring Teaching and Learning for Non-Technical Students", Seventh International Conference on Technology in Collegiate Mathematics, Orlando, FL, November 18, 1994.

24. Honors Colloquium, "Data Driven Mathematics in Economics and the Social Sciences: Using Technology to Model the Real Economic World", University of Rhode Island, Kingston, RI, March 27, 1995.

25. Invited Workshop (co-presented with Iris Fetta), "CALCULUS CONCEPTS: An Informal Approach", Teachers Teaching with Technology Conference, College of Charleston, Charleston, SC, March 4, 1995.

26. Invited Address, "Restructuring Teaching and Learning for Non-technical Students", Teachers Teaching with Technology Conference, College of Charleston, Charleston, SC, March 5, 1995.

27. Invited Workshop, "Business Calculus: Learning in a New Era", Pellissippi State Technical College, Knoxville, TN, February 25, 1995.

John W. Kenelly

1. Science and Mathematics Faculty, Jacksonville University, Jacksonville, FL, September 11, 1992.

2. Louisiana Statewide Systemic Initiative Mathematics Conference, Lafayette, LA, October 2, 1992.

3. NSF Calculus Principal Investigators Annual Meeting, Washington, DC, October 24, 1992.

4. Fifth International Conference on Technology in Collegiate Mathematics, Chicago, IL, November 13, 1992.

5. Mathematical Association of America Annual Meeting, San Antonio, TX, January 15-16, 1993.

6. College Board Southeast Regional Meeting, Hilton Head, SC, February 19, 1993.
7. New Jersey Two Year College Consortium, Kenilworth, NJ, Feb. 27-28, 1993.
8. Montgomery County Mathematics Teachers, Rockville, MD, March 2, 1993.
9. Michigan Calculus Network Conference, Dearborn, MI, March 20, 1993.
10. Mathematical Association of America, Rocky Mountain Section Meeting, Golden, CO, April 3, 1993.
11. Frostburg Technology in Mathematics Conference, Frostburg, MD, April 16, 1993.
12. Conference: Preparing for a New Calculus, Monticello, IL, April 22, 1993.
13. "Technology in Education Conference", Birmingham England, UK, September 18-19, 1993.
14. Florida Council of Teachers of Mathematics State Conference, Jacksonville, FL, October 15, 1993.
15. "Technology In Collegiate Mathematics Conference", Parsippany, NJ, November 5, 1993.
16. "Sigma Xi Lecture on Collegiate Teaching", Florida Atlantic University, Boca Raton, FL, January 27, 1994.
17. "Sigma Xi Workshop on Collegiate Teaching", Florida Atlantic University, Boca Raton, FL, January 28, 1994.
18. "Evening Banquet Presentation", Weber State University, Ogden, Utah, February 11, 1994.
19. "Annual Mississippi School Superintendents Meeting", Jackson, MS, February 21, 1994.
20. National Council of Teachers of Mathematics Regional Meeting, San Francisco, CA, February 24, 1994.
21. Teaching and Learning of Mathematics Conference, Jacksonville, Florida, April 7, 1994.
22. Departmental Colloquium, University of Florida, Gainesville, FL, April 7, 1994.
23. Mathematical Association of America, Kentucky Sectional Meeting, Morehead, KY, April 9, 1994.
24. National Council of Teachers of Mathematics National Meeting, Indianapolis, IN, April 13, 1994.
25. Delta Teachers Academy, Regional Workshop, Caruthersville, Missouri, April 21-22, 1994.

26. Invited Address: "Mathematics in Action. A Simulated Epidemic, Product Marketing and Airplane Flights", Frostburg Mathematics Conference, Frostburg College, Maryland, April 21, 1995.

27. Presentation and Group Discussion: "Teacher Needs For Teaching Mathematics with Calculators", Teacher Inservice Leadership Conference, Fairfax, VA, April 29, 1995.

28. Invited Address: "Technology Needs in the Business Workforce", South Carolina Tech Prep Conference, South Carolina Department of Education, Columbia, SC, May 11, 1995.

29. Invited Address: "New Needs for Today's Students", Collegiate Curriculum Reform and Community Action in the Historic Black Colleges Conference, North Carolina A&T University, Greensboro, NC, May 12-13, 1995.

30. Invited Banquet Address: "Technology Needs in the Business World", Holly Hill Mu Alpha Theta Chapter Installation, Holly Hill, SC, May 15, 1995.

31. Invited Address: "SAT and the Workplace", Clemson-Calhoun Rotary Club, Clemson, SC, May 26, 1995.

32. Ten Day Historic Black College Faculty Professional Development Workshop: "Technology for College Faculty", Spelman College, Atlanta, GA, June 6-15, 1995.

33. Ten Day Tech Prep Faculty Development Institute: "Technology Needs of Business Students", Broward County Schools, Fort Lauderdale, FL, June 19-30, 1995.

34. Multiple Session Workshop: "Workshop on TI-92", Technology Intensive Calculus for Advanced Placement Conference, San Jose, CA, June 25, 1995.

35. Invited Address: "Calculus Topics for Tech Prep Business Students", Southern Regional Education Board Tech Prep Conference, Atlanta, GA, July 13, 1995.

36. Address and Discussion: "Mathematics in Action", Clemson Conference on Business Calculus, Clemson, SC, July 28-30, 1995.

37. Invited Address: "Business Calculus Reform and Technology", Mathematical Association of America Conference for New Faculty, Burlington, VT, August 7, 1995.

38. Invited Address: "Mathematics in Action. A Simulated Epidemic, Product Marketing and Airplane Flights", South Carolina Council of Teachers of Mathematics, Myrtle Beach, SC, November 3, 1995.

39. Invited Address: "Business Calculus Reform", American Mathematical Association of Two Year Colleges Annual Meeting, Little Rock, AR, November 12, 1995.

40. Faculty Workshop: "Calculus on the new TI-92", International Conference on Collegiate Teaching of Mathematics, Houston , TX, November 18, 1995.

Iris B. Fetta

1. "Precalculus and Calculus Using the TI-81 Graphing Calculator", Truett-McConnell College, Cleveland, GA, April, October, 1992.

2. "Using the Graphing Calculator in Teaching Mathematics" , workshop presented at the Changes in Mathematics Education Conference at Northwestern High School, Rock Hill, SC, August, 1992.

3. "Exploring Statistics with Graphing Calculators", special presentation to the Advanced Placement Statistics Task Force, Princeton, NJ, September, 1992.

4. Seventh Annual SCAET (South Carolina Association for Educational Technology) Conference. Session: "Using Hand Held Computers In the Classroom", Columbia, SC, October, 1992.

5-6. The Fifth Annual International Conference on Technology in Collegiate Mathematics, Chicago, IL. Presenter of workshops "Exploring Probability and Statistics with the TI-85" and "Precalculus, Calculus, and Statistics Applications for the Sharp EL-9300"; Copresenter (with Gloria Dion) of Session "Introduction to the Sharp EL-9300", November, 1992.

7. "Using the TI-81 in Precalculus, Calculus, and Statistics Classes", Workshop presented at the South Carolina Teachers of Mathematics Annual Meeting, Hilton Head, SC, November, 1992.

8. American Mathematical Society and Mathematical Association of America Joint Meetings, Presenter of Minicourse #3 "All Right! I've Got a Graphing Calculator. What Happens Next?", San Antonio, TX, January, 1993.

9. "Do I Teach Differently With a Graphing Calculator In Hand?" Workshop presented at the Third Annual Upstate Instructional Technology Conference, Blue Ridge High School, Greer, SC, January, 1993.

10. "Math Wizardry in Everyday Life" - one week course taught to Elderhostel course participants, Clemson, SC, February, 1993.

11. "Teaching with the TI-81 Graphing Calculator", workshop at the Spring Inservice Workshop for Middle and Secondary Mathematics Teachers of Cherokee and Union Counties, Gaffney High School, Gaffney, SC, March, 1993.

12. "Communicating and Testing With Graphing Calculators in Algebra, Precalculus and Statistics", workshop at the Southeastern Regional Conference of the National Council of Teachers of Mathematics, Columbus, GA, March, 1993.

13. "Introduction to the TI-81 Graphing Calculator" and "Effective Use of the Graphing Calculator in the Mathematics Classroom", invited workshops at the City University of New York Faculty Conference, Manhattan Community College, NY, March, 1993.

14-15. "Math Wizardry in Everyday Life" and "Will I Ever Use This?", sessions at the Southeastern Section meeting of the Mathematical Association of America, April, 1993.

16. "Using the Modelling Capabilities of Graphing Calculators to Make Derivatives Intuitive in Business Calculus", workshop presented at ICTMA-6, the 6th International Conference on the Teaching of Mathematical Modelling and Applications, University of Delaware, Newark, DE, August 15-19, 1993.

17. Invited presenter of the high school in-service faculty workshop for Stephens County mathematics teachers "Using the Graphing Calculator in Teaching Mathematics", September 21, 1993, Toccoa, GA.

18-19. Presenter of the two workshops "Calculator Games for the Sharp EL-9300" and "Let's Talk About the Weather" at the North Carolina Council of Teachers of Mathematics Conference, Greensboro, NC, September 30-31, 1993.

20. "Ways to Use a Graphic Calculator in Precalculus, Statistics and Business Calculus", workshop presented to mathematics and science faculty, University of Pittsburgh at Greensburg, PA, October 2, 1993.

21. Invited presenter of the in-service high school faculty workshop (teachers from seven Georgia counties) "Using the TI-81, TI-82, and TI-85 Graphing Calculators to Teach Mathematics", Pioneer Regional Educational Service Agency, October 25, 1993, Cleveland, GA.

22-23. "Business Calculus with the Sharp/TI-82 - Part I" and "Business Calculus on the Sharp/TI-82 - Part II", two workshops presented jointly by Iris B. Fetta and Cindy Harris at ICTCM-6, the Sixth Annual International Conference on Technology in Collegiate Mathematics, November 4-7, 1993, Parsippany, NJ.

24. Presenter of the invited session "Calculator Games" at the South Carolina Council of Teachers of Mathematics Fall Conference, Greenville, SC, Nov. 18, 1993.

25. Presenter of the paper "A Look at the Mathematics in Clive Cussler's Novel *Sahara*" in the Environmental Mathematics Session at the Joint AMS/MAA Meetings, Cincinnati, OH, January 12-15, 1994.

26. "Using the TI-82 Graphics Calculator to Enhance the Teaching and Learning of Mathematics", workshop presented to mathematics and science faculty, College of Charleston, Charleston, SC, January 29, 1994.

27. Invited presenter of the high school/college session "Build Connections Between Probability and Other Math Areas with Graphing Calculator Games" at the NCTM Southern Regional Conference, Richmond, VA, February 23-26, 1994.

28. Invited presenter of the in-service junior high/high school Pickens County faculty workshop "Using the TI-82 Graphics Calculator to Enhance the Teaching and Learning of Mathematics", March 11, 1994, Liberty High School, Liberty, SC.

29-30. "Calculator Graphs: Magic? or Mathematics?" and "Business Calculus II: Modelling Change with Real Data", MAA Southeastern Sectional Annual Spring Meeting, Jefferson City, TN, April 8-9, 1994.

31. Invited presenter of middle school/junior high workshop "Linking Math Topics with Calculator Games Using the Sharp EL-E300 and EL-9300 Calculators", 72nd Annual Meeting of the National Council of Teachers of Mathematics, Indianapolis, IN, April 13-16, 1994.

32. Invited presenter of the in-service faculty workshop "Using the TI-81 Graphing Calculator to Enhance the Teaching and Learning of Calculus", Truett McConnell College, Cleveland, GA, April 23, 1994.

33. Invited presenter of the two workshops "HP-48G Calculator Enhancement for Statistics" and "HP-48G Calculator Enhancement for Calculus", Technology Conference on Using the Texas Instruments and Hewlett Packard Graphing Calculators in the Teaching of Mathematics and Science, Georgia Tech, Atlanta, GA, May 27-28, 1994.

34. Invited presenter (by Educational Testing Service) of workshop "Calculator Enhancement for Advanced Placement Statistics", Introductory Conference for Advanced Placement Statistics Teachers, Clemson, SC, June 10, 1994.

35. Invited presenter of workshop "Exploring Linear Functions with the TI-82", 5th Annual Conference on Technology Focusing on Graphic and Symbolic Calculators, July 8-9, 1993, Houston, Texas, July 8-9, 1994.

36. "Exploring Statistics with the Graphing Calculator" and "Programming your Graphics Calculator", 1994 Summer Sharp EL-9200/9300 Graphing Calculator Conference, Montclair State University, New Jersey, July 22-24, 1994.

37. Invited presenter of session "Calculator Graphs: Magic? or Mathematics?", The Third Carolinas Mathematics Conference, Charlotte, NC, October 6-7, 1994.

38-39. Two workshops (with Don LaTorre): "Business Calculus: Modeling with the TI-82, Part I" and "Business Calculus: Modeling with the TI-82, Part II", Seventh Annual International Conference on Technology in Collegiate Mathematics, Orlando, FL, November 17-20, 1994.

40. "The Guessing Game: A Course-Independent Application of Statistical Reasoning" in the Making Statistics Come Alive Session at the Joint AMS/MAA Meetings, San Francisco, CA, January 4-8, 1995.

41. Invited Workshop (co-presented with Don LaTorre), "CALCULUS CONCEPTS: An Informal Approach", Teachers Teaching with Technology Conference, College of Charleston, Charleston, SC, March 4, 1995.

42-43. Workshops (co-presented with Grace Cascio-Houston) "Exploring the Casio CFX-9800G Color Graphing Calculator", Miami, FL, March 10-11, 1995 and Atlanta, GA, April 22-23, 1995.

44. Presenter of the paper "Modeling the Accumulation of Change = Business Calculus + the Integral + Real Data", Southeastern Section Meeting of the Mathematical Association of America, University of North Carolina at Asheville, Asheville, NC, March 31-April 1, 1995.

45. Workshop (co-presented with Gloria Dion) "Finding Functions in Unexpected Places", NCTM 73rd Annual Meeting, Boston, MA, April 6-9, 1995.

46. Invited presenter of workshop: "Are You Tired of Black and White Solutions?" for South Carolina Mathematics Supervisors, Columbia, SC, April 28, 1995.

47. Invited presenter of the inservice workshop "Linking: Probability, Statistics, and the TI-82" for faculty members in business, psychology, and mathematics at USC-Aiken and Augusta College, Aiken, SC, May 6, 1995.

48. Organizer for and presenter of one of 10 sessions: "The Effect of the Graphing Calculator on the 1995 Advanced Placement Exam in Calculus", Technology Intensive Calculus Advanced Placement in connection with the Harvard Calculus Conference, San Jose, CA, June 23-25, 1995.

49-50. Workshops: "Let's Talk About the Weather!" and "Business Calculus: Modeling and Real Data", Sixth Annual Conference on Technology Focusing on Graphics and Symbolic Calculators, Houston, TX, July 14-15, 1995.

51. "Exploring Statistics with the Graphing Calculator" and "Programming your Graphics Calculator", 1995 Summer Sharp EL-9200/9300 Graphing Calculator Conference, Loyola College, Towson, MD, July 22-24, 1994.

52. Invited workshop: "Exploring Student-Designed Linear Functions with the Graphing Calculator", 5th Annual North Carolina State Mathematics Conference, Greensboro, NC, September 21- 22, 1995.

53-54. "It's As Easy as ABC - Learn to Program Your Graphing Calculator" and "A Rainbow of Mathematics", South Carolina Council of Teachers of Mathematics Annual State Meeting, November 2-3, 1995, Myrtle Beach, SC

55-56. Session: "The Guessing Game: A Course-Independent Application of Statistical Reasoning" and workshop: "Business Calculus: Modeling Multivariable Change", Eighth International Conference on Technology in Collegiate Mathematics, Houston, TX, November 16-19, 1995.

Cynthia R. Harris

1. Michigan Calculus Conference, Dearborn, MI, March 19-21, 1993.

2. Mathematical Association of America Southeast Regional Meeting, Conway, SC., April 2, 1993. Paper presented: "Making Derivatives Intuitive".

3. Second Conference on the Teaching of Calculus, Harvard University, June 25-26, 1993. Paper Presented: "Modelling in Business Calculus".

4-5. "Business Calculus with the Sharp and TI-82 Calculators, Parts I and II", two workshops presented jointly by Cynthia R. Harris and Iris B. Fetta at the Sixth Annual International Conference on Technology in Collegiate Mathematics, November 4-7, 1993, Parsippany, NJ.

6. "Student Projects in Business Calculus", presentation to the Curriculum Committee, College of Commerce and Industry, Clemson University, March 1, 1994.

7. "Teaching Methods and Focus for Calculus Concepts: Chapters 1-5", Clemson Conference on Business Calculus, Clemson, SC, July 28-30, 1995.

8. "Business Calculus: Modeling with Real Data", a workshop co-presented with Laurel Carpenter, Eighth International Conference on Technology in Collegiate Mathematics, Houston, TX, November 16, 1995.

Other Project Presentations:

1. Laurel Carpenter, Clemson University: "Modeling Change = Business Calculus + the Derivative + Real Data", MAA Southeastern Section Annual Meeting, Asheville, NC, March 21, 1995.
2. Laurel Carpenter, Clemson University: "Business Calculus: Modeling with Real Data", a workshop co-presented with Cindy Harris, Eighth International Conference on Technology in Collegiate Mathematics, Houston, TX, November 16, 1995.
3. Paul Ache III, Texas A&M University: "Mathematical Modeling, Business Calculus, and the Graphing Calculator", Eighth International Conference on Technology in Collegiate Mathematics, Houston, TX, November 19, 1995.

APPENDIX II

Publications

relating to the project and/or its products

Published Papers

Donald R. LaTorre

1. "Business Calculus: Restructuring Teaching and Learning for Non-technical Students", *Proceedings of the Seventh International Conference on Technology in Collegiate Mathematics*, Addison-Wesley, 1995.

Iris B. Fetta

1. Dion, Gloria S. and Iris B. Fetta. "Calculator Graphs: Magic? or Mathematics?", *Mathematics and Computer Education* 27(3) (Fall 1993): 180-197.
2. Dion, Gloria S. and Iris B. Fetta. "Calculator Graphs: Magic? or Mathematics?", Spanish translation appearing in *Las calculadoras graficadoras en el aula,* Departamento de Mathematica Educativa, CINVESTA, June 1994.
3. "Precalculus, Calculus and Statistics Applications for the Sharp EL-9300", pages 130-134 and "Exploring Probability and Statistics with the TI-85", pages 301-304, *Proceedings of the Fifth Annual International Conference on Technology in Collegiate Mathematics*, Lewis Lum, ed., Addison Wesley Publishing Company, 1994.
4. "Programming the Sharp EL-9200/9300C Graphing Calculator", a handout given to teachers using this calculator in the mathematics classroom, 1994-95.
5. Co-developer of "Activities for the Casio CFX-9800G Color Graphing Calculator", activities published for use by workshop participants and put on the WEB for general access by teachers, 1995.
6. Dion, Gloria S. and Iris B. Fetta, "Everybody Talks About It!" *The Mathematics Teacher* , National Council of Teachers of Mathematics, February, 1996.

Cynthia R. Harris

1. "Exploiting the Modelling Capabilities of Graphing Calculators in Business Calculus", *Mathematics and Computer Education* 27(3) (Fall 1993): 226-232.

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Accepted Papers

About the project:

"Clemson University's Calculus Concepts: An Informal Approach to the Mathematics of Change, Preliminary Edition", *A Systemic Description of Eighteen of the Calculus Reform Projects Written in Recent Years*, a product for the Calculus Reform Work-shops conducted under Don Small's and Wayne Roberts' NSF project, M. Kathleen Heid, editor, to appear, April, 1996.

Iris B. Fetta

"The Guessing Game: A Course-Independent Application of Statistical Reasoning", *Proceedings of the Eighth International Conference on Technology in Collegiate Mathematics*, Addison-Wesley, to appear, November, 1996.

Published Books

By the Project:

1. Calculus Concepts: An Informal Approach to the Mathematics of Change, Preliminary Edition, D. C. Heath and Company, August 1995.
2. Calculus Concepts: An Informal Approach to the Mathematics of Change, Preliminary Edition Multivariable Chapters, D. C. Heath and Company, January 1996.

Iris B. Fetta

1. Lawrence, David P. and Iris B. Fetta, Applying Statistics using the Sharp EL-9200, El-9300 Graphing Calculators, Pencil Point Press, 1994.
2. Lawrence, David P. and Iris B. Fetta, Applying Precalculus/Calculus using the Sharp EL-9200, El-9300 Graphing Calculators, Pencil Point Press, 1994.
3. Linking: Probability and Statistics with the TI-82, Pencil Point Press, Fairfield, NJ, 1995.
4. Graphing Calculator Instruction Guide to accompany Calculus Concepts: An Informal Approach to the Mathematics of Change, Preliminary Edition, D. C. Heath and Company, August 1995.

Laurel L. Carpenter

1. Instructor's Guide with Complete Answer Key to accompany Calculus Concepts: An Informal Approach to the Mathematics of Change, Preliminary Edition, D. C. Heath and Company, August 1995.

Books Pending

By the Project:

1. Calculus Concepts: An Informal Approach to the Mathematics of Change, Single Variable Chapters, Houghton-Mifflin Publishing Company, to appear, August 1997.
2. Calculus Concepts: An Informal Approach to the Mathematics of Change, Single and Multivariable Chapters, Houghton-Mifflin Publishing Company, to appear, August 1997.

Iris B. Fetta

1. Graphing Calculator Instruction Guide to accompany Calculus Concepts: An Informal Approach to the Mathematics of Change, Houghton-Mifflin Publishing Company, to appear, August 1997.

Laurel L. Carpenter

1. Instructor's Guide with Complete Answer Key to accompany Calculus Concepts: An Informal Approach to the Mathematics of Change, Houghton-Mifflin Publishing Company, to appear, August 1997.

APPENDIX III

External Evaluation Appendix

EVALUATION APPENDIX

INDEX

III-A Institutions Who Will Field Test in 1995-1996

III-B Sample Discussion Materials

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APPENDIX III-A

1995-96 Field Test Institutions

(Summer Conference Attendees)

Institutions Who Will Field Test in 1995-96

Anderson College, SC
Arizona State University, AZ
Bob Jones University, SC
Boston University, MA
Bradley University, IL
Clemson University, SC
College of the Siskiyous, CA
Drake University, IL
Elmhurst College, IL
Greenville Technical College, SC
Kennesaw State University, GA
Lenoir-Rhyne College, NC
Manchester Community College CT
Oklahoma State University, OK
Pellissippi State Technical Community College, TN
Raritan Valley Community College, NJ
Santa Barbara Community College, CA
Texas A&M University, TX
University College of the Fraser Valley, CANADA
University of Colorado, CO
University of South Carolina-Aiken, SC
University of Texas-Arlington, TX
University of Vermont, VT
Villanova University, PA
Xavier University, OH

(Editorial Note: Easley High School, SC and Staples High School, CT are known to be field testing this year but did not have representatives at the conference. D. C. Heath estimates that approximately 25 other schools are also using the materials in pilot sections during this year.)

APPENDIX III-B

Classroom Observations: Summer 1993

- Project I: Summer Session 1993 Student Samples
- Final Project: Summer Session 1993 Student Samples
- Student Interview Summaries: Summer Session 1993

Class Observations: Summer 1993

It was decided that Ms. Searcy would sit in on this class for its duration as a participant observer. She would record notes on her observations, collect sample student work and interview both staff and students. Ms. Searcy arrived a couple of days before the class began and had several in-depth conversations with Don LaTorre, Cindy Harris, and Iris Fetta about the progress that had been made to date. Professor Kenelly was out of town, but would return later in the week. For various topics, Ms. Searcy offers some excerpts from her notes that give a flavor of the discourse and events that happened during that Summer session course. In these notes, first person comments refer to Ms. Searcy.

As one can immediately discern from the Business Calculus I text that has been produced by the project staff, mathematical modeling is considered a very important tool in the revitalization effort. The introduction of this tool into the Business Calculus curriculum brought some interesting challenges to students' concepts of mathematics.

> July 1, 1993:
>
> (Note: Cindy Harris substituted for John Kenelly 6/30 - 7/2.)
>
> ... The first section (of the new written materials) is based on the concept of the mathematical model. The materials did not give an explicit definition for this, so Harris asked the students to tell her what they thought a model was. One of the students said it was something you can use to tell what happened in the past. Professor Harris agreed with him saying that "something" was usually mathematical equations. In this class we will be looking at actual data to form models to predict what will happen in the future. Professor Harris gave the following example. Suppose we wanted to look at the world wide spread of the AIDS virus form 1982 to 1992. The graph would look something like this (Figure 1):

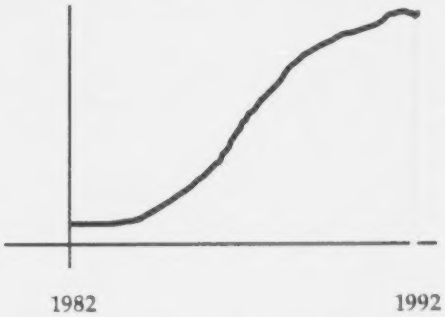


Figure 1: Spread of the AIDS virus

In the last few years, it looks like the spread of the disease is slowing down somewhat. Professor Harris asked what we thought would happen from now until the year 2000. Could the graph look something like this (Figure 2)? Probably not, unless a cure was found.

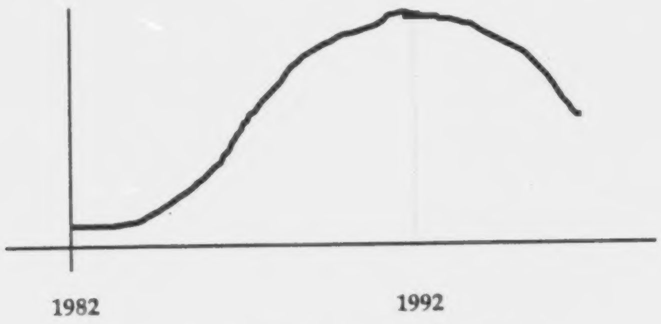
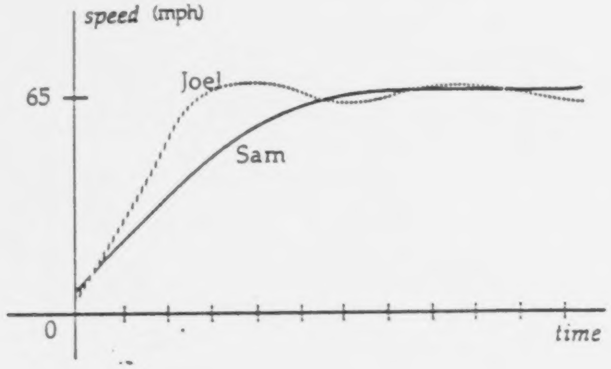


Figure 2: AIDS virus growth possibility

It was stressed that when considering the composition of a model for a situation, there are usually a lot of factors to contemplate. (We looked on page 2 of the materials at a couple of examples.) However, in this class, we will not use all these factors in our model to keep the equations from getting too complex. Another important point became obvious in the car example on page 3. We were looking at the graph and tried to decide what the scale should be for the time axis. We did this by discussing how long it took a car to go from 10 mph to 65 mph. Some students said 3 seconds, while others said 1, 4, or 5 seconds. Professor Harris pointed out that all these answers were correct depending on what car one was driving. She explained that there will be several times in this class where you may not have the same answer as everyone else. But that's fine. There won't always be an exact right answer. Mathematics isn't always exact.

Modeling also challenges students to think about the situation and rational behind certain models. In the next excerpt, students are asked to think about a couple of applications of the linear model.

One of the first steps in modelling a situation is an understanding of the facts that are known. This information is often presented in the form of a graph. Consider the following graph showing the speed of two cars on an interstate highway:



What general information is presented by this graph?

It appears that Joel and Sam are the drivers of the cars (or the cars could be named Joel and Sam and driven by two persons whose names we do not know). We also can probably assume that 65 mph is the speed limit on this interstate highway.

Which car appears to be accelerating faster?

Since Joel's entrance speed is slightly lower than Sam's and Joel reaches the speed limit earlier than Sam, Joel's speed must be changing at a faster rate than Sam's.

Has either car gone faster than the speed limit?

Yes. Imagine a horizontal line 65 units above the *time* axis. Joel's speed crosses this line several times.

Interpret the value time = 0 in the context of this problem situation.

Since this is an interstate highway with a minimum speed requirement and no traffic lights, the speed of each car when time equals 0 probably represents the speed at which each car is travelling when it turns onto the entrance ramp for the highway or the speed at which the car leaves the exit ramp and enters the flow of traffic.

Notice that no units have been placed on the *time* axis. Based on your own driving experience, give a numerical value and unit of measure for the first "tic" mark on the horizontal axis.

This value will, of course, differ for each person and type of vehicle the person is driving. It appears to take Joel approximately 2.5 time units to reach 65 mph while Joel more leisurely approaches that value. In order to further discuss this problem, let's just say the first "tic" mark on the horizontal axis represents 4 seconds.

Which car appears to be using cruise control?

The graph indicates that Sam's speed increases for approximately 28 seconds and then remains constant. Sam is probably using his cruise control.

Will Sam pass Joel?

There is not enough information given in the graph to answer this question. We are not told whether or not these two vehicles are travelling in the same direction. We do not even know if they are on the same portion of the interstate.

In this course you will often be using your calculator for evaluating numeric expressions, solving equations, estimating and checking the reasonableness of algebraic expressions, and graphing mathematical models. Let us now consider how the power of the graphing calculator can aid in your exploration of some other practical situations illustrating the use of mathematics and modelling in today's world.

Example 1: A stockbroker's client purchases 250 shares of stock at $29.75 a share. Two months later, the client sells the stock at $43.50 a share. If the stockbroker's fee is $45 per transaction, what is the client's net profit?

Solution: The model the stockbroker uses to determine the client's net profit is given by the equation *net profit = selling price – (purchase price + transaction fees)*. Let's use the REAL MODE of your Sharp 9300 for the calculations. Press $\boxed{\substack{+\,-\\ \times\,\div}}$ to access the REAL MODE, and enter 250 $\boxed{\times}$ 29.75 to obtain $7437.50 as the purchase price of the stock. Compute the selling price of the stock as 250 $\boxed{\times}$ 43.50 = $10,875. The transaction fee is fixed at $45; that is, it remains the same regardless of the number of shares of stock that are bought or sold. The client must therefore pay $45 when purchasing the stock and another $45 when selling the stock. The client's net profit is computed from the from the equation to be

$10,875 – ($7437.50 + 2*45) = $3347.50.

Example 2: The following table lists the accumulated amount A in a savings account earning 5% simple interest yearly. The variable t represents the number of years since the account is opened. Assume that the initial deposit is $1000 and that no other deposits or withdrawals are made from the account.

t	0	1	2	3	4
A	1000	1050	1100	1150	1200

July 8, 1993:

Today began with the students voting to have their first test on Tuesday. We then proceeded to look at problem #4, page 25. Professor Kenelly asked the students why prices take on a linear form. He answered that companies think that they can slip another dollar in every year or two. Thus they have linear growth. Example: Ford Mustangs. How much price change (for a Ford Mustang) occurs per year? One student said $300 (because people wouldn't go for it if it were too high) and another said $2000 (they have to keep up with newer cars coming on the market -- they have to make improvements.) Mustang prices increase $500 every year. Why? Because they have a good product and they sell consistently. Manufacturers can't afford to lose popularity so they can't raise prices much. What's the best way to know what to increase by? The answer is "Did it work last year?" This is called momentum. You need to know models and be able to look at the data.

... (Back to the problem.) Does it (the data) look linear on a scatter plot? We did a regression line to fit the data -- looked like a good fit. Professor Kenelly asked students what a football ticket is worth. Student answers: 1) Different amounts to each person and 2) Whatever the fans will pay. There is no product being sold here. What is being sold? Student answer: Entertainment. Why did they increase form $8 to $9? Student answer: Cause they sold a lot of tickets last year. It worked so do it again. Another student said that it was "justified." Professor Kenelly told him that there was no justification to it, it just worked. Now everyone says, "Do it again next year!" Suppose you raise it two dollars instead of one? If it worked, good (promotion). If it doesn't work, you get fired. A couple of reasons to say two dollars is because of an aggressive nature and because you looked at your model and saw that with the model you could raise prices $2. (This last point came in a roundabout way.)

We then looked at the equation of our best fit line. It was

$$y = -47.8 + 0.72x.$$

Is this model guaranteed? Students answered that it wasn't. Kenelly said that they had done some good thinking, and there is no guarantee on the model. After some discussion of the correlation coefficient r, they looked at predictions and decided that it would be shaky for a prediction for 1997 and really shaky for a prediction for the year 2007.

d. Would it be wise to use this model to predict the enrollment in the year 2000? Use your estimate of the 1993 enrollment in an explanation of why you answered as you did.

4. The price of tickets to Clemson home football games is given in the table below.

Year	'77	'79	'81	'83	'85	'87	'89	'91
$ Price	8	9	10	12	13	15	16	18

a. Find the best fitting linear model.

b. Find r and r^2 for your model. Interpret the meaning of each of these in the context of this problem.

c. Use the model to predict the ticket *price* in 1993, 1995, 1997. (Round the nearest $.)

d. Suppose you graduate, get married, and have a child who attends Clemson. Predict the *price* you will have to pay to attend a football game with your child during his or her freshman year. Describe in detail how you arrive at your answer.

e. Repeat part *c.* assuming you have a grandchild who attends Clemson.

5. The percentage of funding for public elementary and secondary education provided by the federal government is given below:

Year	'81-82	'82-83	'83-84	'84-85	'85-86	'86-87	'87-88
% Funding	7.4	7.1	6.8	6.6	6.7	6.4	6.3

a. Find the best fitting linear model.

b. Find r and r^2 for your model. Interpret the meaning of each of these in the context of this problem.

c. What is the rate of change of the model you found?

d. Based on the model, estimate the % of funding provided by the federal government in the 1993-94 school year. Do you think the estimate is reasonable? Why or why not?

6. Consumer credit was $619.7 billion dollars in 1987 and $748.3 billion dollars in 1990. Assume that consumer credit increases at a constant rate.

a. Find the rate of increase.

b. Based on the rate of increase, estimate what consumer *credit* will be in 1994.

As mentioned before, assessment needed to reflect what was happening in the classroom. So much of the modeling section required students to make characterizations of situations. Professor Kenelly's first pop quiz was the simple question "What do you mean by line?" Students were give a few minutes to answer this on their own then Professor Kenelly asked them to get into pairs and give a joint description of "line". This went fairly well; however, it took some students a few moments to understand that he wanted them to give characteristics of linear models.

The major assessment task for the modeling section came in the form of a group project. The groups each consisted of three students. Each group member had a specific responsibility that was assigned to them by Professor Kenelly. These responsibilities were 1) Scribe: keeps minutes of the group meeting, 2) Organizer: makes arrangements for meetings, and 3) Reality Checker: responsible for making sure everyone understands one another. They were assigned a project which required them to examine the current student fee refund policy of Clemson University. (See the insert on the following page.) They were to look at alternative models and choose one that would satisfy the largest percentage of stakeholders. They had to explain why they accepted or rejected a specific model. A couple of student samples from the Summer Session 1993 projects can be found at the end of this Appendix.

Looking at a list of data may be fun for some students, but actually creating data was an interesting experience for the entire class one day.

> July 16, 1993:
>
> ...The class participated in a rather interesting activity. There were 26 students in the class. The students were numbered from 1 to 26. Using the graphing calculator's random number generator, we chose one person at random to enter the classroom with flu at 8 a.m. and sneeze. (This student was then to go to the back of the classroom since he had the flu.) Then, at 8:10 a.m. another person catches the flu from the first and we generated a random number to represent this occurrence and determine this other person. That person went to the back of the room as well. These two people then sneezed, each giving the flu to another person. Two random numbers were generated to represent these people. Again, the students with those numbers went to the back of the classroom. When a number was repeated, we considered it as the event when no one new was infected by the sneeze. The following is the data that we generated.

MTSCH 102 PROJECT I
Clemson University
Summer II 1993

SETTING: Recent budget cuts have forced universities in South Carolina to cancel many required classes and students are having to repeatedly enroll and withdraw from the University in attempts to find the classes that they need. The fee refund policies are being questioned and a great deal of public debate is taking place. The Higher Education Commission has scheduled hearings on the issue this fall and the Clemson Board of Trustees has hired your firm as consultants to help them prepare their presentation. The student senate has passed a resolution condemning the current schedule. The faculty senate has appointed a committee and asked for a meeting with the President's cabinet. The Vice President for Business has hired a new Associate Vice President for Student Fee Refunds and he has authored an article in the *Greenville News* stating that further erosion of the University's ability to retain student fees would cause even fewer course offerings.

TASK: Examine the current fee refund schedule. Critique it and explain everyone's displeasure. Examine alternative plans, including at least the linear, quadratic, exponential, square root, logarithmetic, no refund and complete refund models. Detail your full list of alternatives and explain the resulting schedules that each would produce. Make a selection and justify your choice. Give an explanation for each alternative plan that you have not taken and justify your rejection. Prepare the board's response to any member of the commission when the individual asks the Clemson representative why the University did not accept any one of the alternative plans. (The commission has farmers, businessmen, parents, plant managers, educators, and politicians in its current membership.) Prepare the press release that Clemson University will use when it announces the installment of your plan. (Remember press releases should attend to the What, When, Where, Why.and Who

Table 1: Spread of an Epidemic Data

Time	Number Infected
8:00	1
8:10	2
8:20	4
8:30	7
8:40	9
8:50	16
9:00	23
9:10	25
9:20	26
9:30	26

We plotted the data and got a logistic type curve. We then ran the "Logistic 1" program with limiting value 27. The "Logistic 2" program then graphed this function for us. Using the real mode of the Sharp EL-9300 calculator, we found the coefficients for the logistics equation

$$y = L/(1 + Ae^{-Bx}).$$

They were $A = 63.0179$ and $B = 0.78$. We also got the values $I = 5.26$ and $J = 13.5$. These were explained to be the coordinates of the inflection point.

Professor Kenelly asked why anyone would want to know this equation. A student said that it could be used to stop the epidemic. Kenelly said that we can't really stop it. Another student said we could use the equation to be prepared for the epidemic. Professor Kenelly agreed and said that the infirmary would need to know how many beds they would need. We then looked at the first part of the data up to 8:40 a.m. We ran the logistic program just on this section. We found $A = 46.38$, $B = 0.66$, $I = 5.8$ and $J = 13.5$. From the graph of this new model, we saw that we could have predicted pretty close the remainder of the data.

When does it (the model) break? At $(I,J) = (5.8, 13)$. We also looked at different values of x and saw approximately how many people have contracted the flu. At $x = 8$ (around 9:10 a.m.) we got $y = 21.84$ or approximately 22 people have the flu. What else is the model good for? Making health plans by understanding models. What would happen if two people came in with the flu, instead of one? What if we took half of the people with the flu out of the room? Would it have slowed down?

Students also looked at other types of models. Professor Kenelly often wanted students to characterize these models in every day language and not just rely on mathematics jargon. This often frustrated students because they were not used to this type of explanation process.

> July 22, 1993:
>
> ...Next John asked students to explain the difference between the exponential and logistics models. One student answered that logistic curves have inflection points. Professor Kenelly asked what were inflection points. Students answers were 1) where rate changes, 2) where it goes from concave up to concave down, and 3) where rate goes from increasing to decreasing. With each response John said, "Don't give me none of that math stuff!" He then proceeded to say it was a change in behavior. It has different characteristics on each side of that point.
>
> Students were asked to differentiate between logistic and exponential curves. A difference given by one student was that one slows down and the other keeps going up. (Professor Kenelly offered that the exponential goes up "eternally".) He related this to the birth rate of Mexico City. He asked what kind of model it was following now. A student said that it was exponential. Why can't it go on forever? Another student said that there is not enough room. Kenelly agreed and added that there is also not enough jobs or food.
>
> Finally, a last difference was given by yet another student. Logistics curves approach a limit. Professor Kenelly said a better word would be maximum or saturation point.

Because of the newness of the material, it was difficult to judge just how much time should be spent on each topic. Thus, by the end of the session, Professor Kenelly was pressed for time to get into the topic of derivatives. It also seemed that he was not quite as sure of himself in presenting material on rates of change and tangents. He strived to stay with the same philosophy that these topics can be communicated without using so much mathematics jargon. This took a great deal of thought and time to carry out. However, it was always helpful to relate concepts back to applications.

July 30, 1993:

...We graphed *y =sin(x)* and *y = cos(x)* (Figure 3). We were told that *y = cos(x)* is the rate curve of *y = sin(x)*.

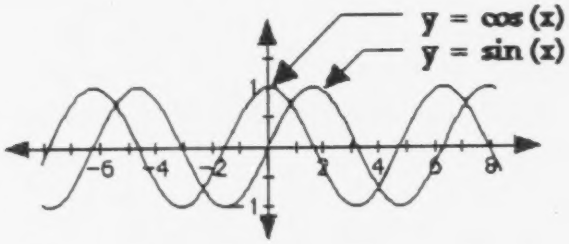


Figure 3: Graphs of the sine and cosine functions

Professor Kenelly pulled out an overhead of a stock market report from the Wall Street Journal. In order to study an economic curve you should study its rate curve (its "speedometer"). We discussed positive (where the market increases) and negative (where the market decreases) slopes (of tangent lines.)

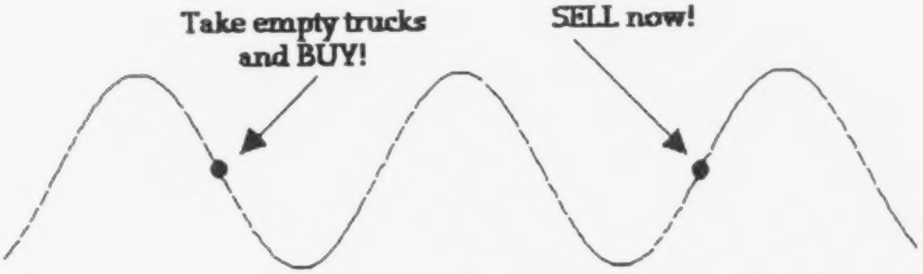
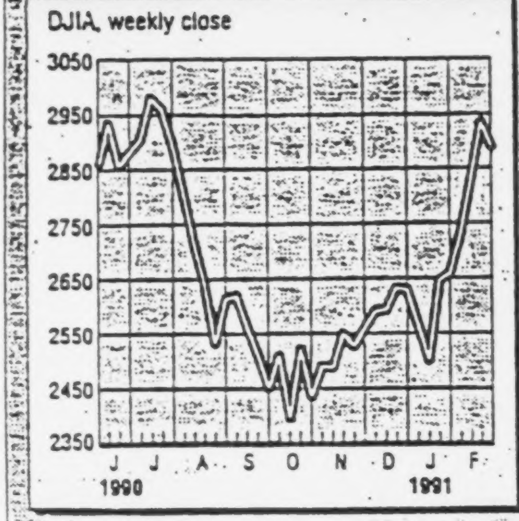


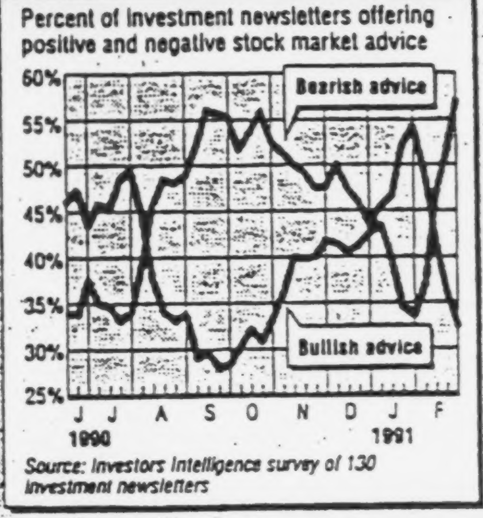
Figure 4: Stock Market Application--Rates of Change

When do you change from selling to buying? When the decreasing rate stops dropping. A student said that happened where the line is (indicating the left most arrow in Figure 4). Another student asked that if its that easy, why aren't we all millionaires? Professor Kenelly said that was a good question. Does everything conform to scientific models? The class said no. Things are not that easy. What does this tell us -- the underlying principle behind that phenomena? Should we study this at all? Student answers were 1) it behaves to some degree that way, and 2) not so much the phenomena curve as the rate of change curve.

As the Market Rises ...



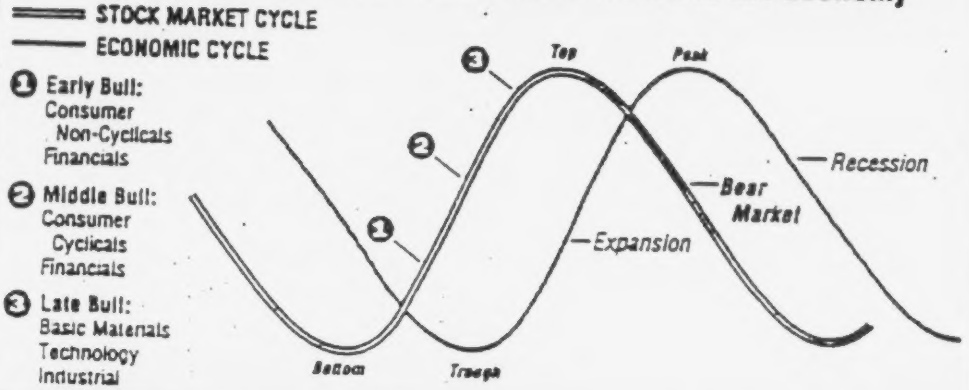
Sentiment Becomes Bullish



Source: Investors Intelligence survey of 130 investment newsletters

Will the Stock Market Follow the Script?

Bull Markets Tend to Have Three Phases Linked to the Economy


STOCK MARKET CYCLE
ECONOMIC CYCLE
① Early Bull: Consumer Non-Cyclicals Financials
② Middle Bull: Consumer Cyclicals Financials
③ Late Bull: Basic Materials Technology Industrial
③
②
①
Top
Peak
Recession
Bear Market
Expansion
Bottom
Trough

Is the Market Moving Into the Third Phase?

Early Bull Market	Middle Bull Market	Late Bull Market
Oct. 31, 1990 to Dec. 20, 1991	Dec. 21, 1991 to June 30, 1992	Began July 1, 1992

> What if you don't have a rate curve? *The Wall Street Journal* doesn't always give the rate curves. One student said that you have the data. Professor Kenelly said that was right. You can look for decreasing rate of the increase in the second differences. In the cycle situation, look at the differences. You need daily information. What drives cycles? One student said advertising. The magic word is "externally" driven. It is not like the logistic curve that feeds on itself. A student then asked, "Can you use cycles to plot how productive a worker is during different shifts?" Professor Kenelly replied, "Sure." (Out of time.)

The course ended with groups (same ones as before) giving presentations on a project that dealt with rates of change and tangent lines. This was not a trivial project, and for the most part, groups did rather poorly on it. When questioned about specific points of their presentation, students often stammered and revealed that they did not exactly understand what they had done. However, there were some groups that did very well with the project and handled questions from the project staff effectively. Examples of the Final Project Reports from the Summer Session of 1993 can be found at the end of this Appendix.

Aside from the projects, Professor Kenelly gave two major quizzes. These were basically given to see how well students were working with the calculators and interpreting data. Students were not allowed to work on these cooperatively. The first test was given while Professor Kenelly was away at a conference. Ms. Searcy proctored this exam.

> July 13, 1993:
>
> ... I looked over the students' papers before I put them in Professor Kenelly's box. The calculator sections seemed to be fairly easy for most students. The discussion questions were better and worse than I expected. Some students did some great thinking. There were several cases, however, where the students just repeated words they memorized from Professor Kenelly's lecture rather that thinking for themselves. Old habits die hard! Students seemed to pick up well on the concepts of exponential and logistics modeling. Professor Kenelly did tell the students that they would not be responsible for logarithmic models, yet the last question dealt with this idea. I encouraged students to answer it to the best of their ability. Several did well on it.

MTSCH 102 PROJECT III
Clemson University Summer II 1993
Rates of Change/Tangent Lines

SETTING: You have been employed by First Trustworthy Bank as a consultant to recommend guidelines for using the "rule of 72" by its employees. Dramatic interest variations have been induced by vacillating economic policies of the government and bank customers are frequently questioning tellers about the growth effects of different CD rates. The bank wants to instruct the employees on how to explain to the customers "years to double" comparisons and it wants to have them use the "rule of 72." Thus, the officers need to know the accuracy of the "rule of 72" and guidelines on its use. You are to brief the officers on those points as well as comment on the sensitivity of the error in the rule to interest rate changes. That is, you need to compare the slopes of the tangent lines on both curves at interest rate levels at the integer values between 2% and 20%. With this data, you should explain what errors in the approximation and differences in the slopes of the two curves mean in relation to improving or degrading accuracy in the approximation. The curves in question are the comparisons of the rule of 72 and the true values for "years to double" as a function of interest rates.

TASK: You are to prepare a draft of the memorandum that will be distributed to the employees in the bank's regular series of prepared answers to customer questions *"When you are asked...."* The draft memorandum should be delivered to the editor on Friday July 30 and as consulting firm #10, your 5 minute presentation to the officers is scheduled at 8:40am on Wednesday Aug 4 in M-104. Other consulting firms will be making their presentations, so the schedule is very rigid. Arrive 10 minutes early and remain in the hall. You will be called in when the officers are ready. The bank's chairman, Don LaTorre and its president, Cindy Harris are available for limited questions at 656- 3437 and 656-1302 respectively. The editor of the *"When you are asked..."* series, John Kenelly, is your principal contact and he can be reached at 656-5217. A Sharp 9300 presentation computer and an overhead projector will be available in the presentation room. You will need to arrive with you presentation transparencies prepared. Your *memorandum* should include and highlight a "caveat" statement for the tellers to make when they respond to customers. This is to protect the bank from legal consequences of its employees using mathematical approximations.

TECHNICAL BACKGROUND:
"The Rule of 72" The numbers of years that it takes money to double is estimated by dividing 72 by the annual interest rate, p, expressed as a percent.

The correct mathematical answer is given by N when the expression, (1+p)^N, equals to 2.

Quiz 1 - Thursday, July 13, 1993 page 1 NAME:____________________________
MTHSC 102 Clemson University

1. (a) Enter the "price of eggs" in China data into your calculator:

80	82	84	86	88	90
2	6	14	22	26	30

(b) Do an auto scatter diagram. Sketch the resulting screen

(c) Do an **ax+b** regression. Sketch the resulting screen.

(d) Which data points are not on the line?

(e) What is the equation of the regression line?

(f) What is r? r^2? Explain what r^2 describes.

(g) Estimate the price at 83.

(h) Estimate the price at 95

2. Use the solver to solve $y = y = Ax^2 + Bx + C$
(a) with $A = 3$, $B = 2$, $C = 1$, $x = 1.234$

(b) with $A = 3$, $B = 2$, $C = 1$, $y = 5$

(c) with $A = 1$, $B = 2$, $C = 3$, $x = 4.321$

Quiz 1 - Thursday, July 13, 1993 page 2 NAME:____________________________
MTHSC 102 Clemson University

3. Compare the graph of $y = x^2$ and $y = x^4$

(a) between $x = 0$ and $x = 1$. Sketch the screen. Show your screen size.

(b) between $x = 1$ and $x = 2$ Sketch the screen. Show your screen size.

4. Explain why your water bill would be a linear function of the number of days in the billing cycle.

5. Explain why the number of pump strokes on a bicycle pump would be a quadratic function of the pressure level you want to achieve.

6. Explain why the gate receipts at a football game are a linear function of paid attendance.

7. Explain why the number of sales on the latest style T-shirt grow exponentially.

8. Explain why advertising effectiveness grows logarithmetically as you increase your ad purchase.

The second exam was basically calculator computation. Students had some difficulty with the RANGE function on some graphs. Also, they had difficulty finding the regression equation coefficients. Several students just put the generic formula for their answer.

Finally, Ms. Searcy wanted to get a feel for how students perceived their experience during this trial run of the course. Ms. Searcy conducted several student interviews. Summaries of these interviews can be found at the end of this Appendix. For the most part, students were pleased with the course and what they learned. They were particularly interested in the applications that were given in the class. One other indication of students' feelings toward the course was given at the end of the session in class.

> August 2, 1993:
>
> ... (Professor Kenelly asked) "Is the class worth it?" One student replied that it is a good program. It needs time to make changes. Students have the attitude that math class is where they watch the teacher write on the board. Time is needed to change students' attitudes. It is worth it! We need it. Board meetings are going to require this kind of stuff. Students need to think on their own. As for group dynamics, we need to implement this at earlier levels.

Quiz 2 - Friday, July 23, 1993 page 1 NAME:______________________________
MTHSC 102 Clemson University

1. (a) Enter the "price of eggs" in China data into your calculator:

1	2	3	4
8	22	60	164

(b) Justify that an exponential model would fit the data?.

(c) Do an auto scatter diagram. Sketch the resulting screen

Xmin = Xmax= Ymin = Ymax =

(d) Do an **a*e^bX** regression. Sketch the resulting screen.

(e) What is the equation of the regression equation?

(f) Estimate the price at 5.

2. Assume that the data in question 1 covers the first four months of the year.
(a) Find the average rate of change over the first two months.

(b) Find the average rate of change over the first four months.

3. (a) Enter the revised "price of eggs" in China data into your calculator:

1	2	3	4	5	6	7	8
8	22	60	164	430	1000	2000	3500

(b) Do an auto scatter diagram. Sketch the resulting screen

Xmin = Xmax= Ymin = Ymax =

(d) Fit a logistic curve. Use L=4000. Sketch the resulting screen.

(e) What is the logistic equation?

(f) Estimate the price at 5.5.

4. Graph the curves Y1 = e^X,Y2 = ln X, and Y3 = -X

Use Xmin = -2, Xmax = 2, Xscl = 1, Ymin= -2, Ymax = 2, Yscl = 1

(a) Sketch the screen.

(b) Find the value of Y' where Y1 = e^X crosses Y3 = -X

(c) Find the value of Y' where Y2 = ln X crosses Y3 = -X.

Project 1: Summer Session 1993

Student Samples

Two Samples Given:

- Group 3
- Group 7

Group # 3

Kim - Scribe	225-9797
Derrick - Organizer	882 - 7732
Stephen - Reality Checker	235 - 6414

1st project - 21st
2nd project - 27th
3rd project - 30th

Minutes: July 15

Discussed dates inwhich we would like projects due. where we would meet, and on what dates we would work on the project. After class we went to the library, registrar's office and student development to get information on the dropout rates. We found that withdrawl rates were not counted or put into the 20%, 40% ect. - catagories

Minutes: July 16

Meeting at Derrick's house

Discussed project step by step and then went through and made a rough draft.

Minutes: July 20th

Discussed after class any changes we would like to make before the final copy would be made.

PRESS RELEASE

On Wednesday, July 21, Clemson University plans to put into effect the new refund schedule for 1994. Recent budget cuts have forced the University to cancel many courses required for graduation and students are forced to repeatedly enroll and withdrawl to find the classes they need. The debate over the current schedule came about when many students felt the University was not fair in the amount of refunds. Clemson University's vice president for student fee refunds had stated that any more cutbacks would further the school's economic problems and cause even less courses to be offered.

The board of trustes' new proposal seems to satisfy both sides. The old schedule decreased the amount refunded by 20% for every week the student attended. The new schedule does not change by a constant rate. Instead, the rate ranges anywhere from 4% to 8% depending on the week. Also, the withdrawal period is lengthened from four weeks to six weeks to allow the students more time to become aquainted with the course.

To make up for the new rate cut, the trustees have placed a 15% buffer on tuition prices. The University keeps this 15%, no matter when the student withdraws, to cover registration costs and other expences. Also in order to keep students from taking advantage of the six week system, extra withdrawal hours are charged to those students who withdrawal after four weeks.

After numerous studies, the University Board Of Trustees found this plan to be most beneficial to all sides. While providing with a new found freedom, this plan also helps Clemson deal with state and federal cuts.

Clemson University Board of Trustees Presentation on the Proposed Refund Schedule Changes*

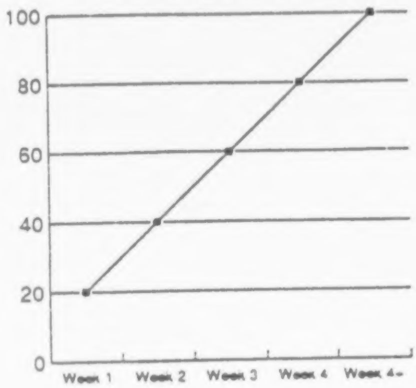
Linear Model – The current schedule allows for a refund percentage of 20% to be removed every week of the class during the first four weeks of attendence. This schedule is used during the Spring and Fall semesters only. Arguments to support both student and faculty views are as follows:

- Many students feel a four week period is inadequate in becoming aquainted with a 15 week course.
- Another objection relates to the rate of increase. By the time the first exam in a class is taken, two to three weeks have already passed; the rate is too high to consider withdrawal.
- After four weeks, a student is denied any refund, even though having only attended for less than a third of the semester.

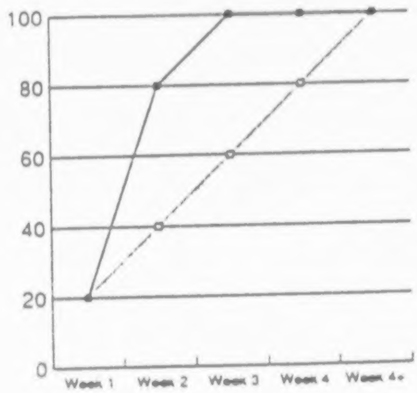
- The University is already facing economic setbacks, any further reductions would cause a decrease in the quality of classes.
- Many faculty members feel that the refund schedule is adequate to compensate for the invested time and effort of professors.
- The University has numerous expenses that accompany each student, even those who withdrawal. There are registration costs, parking management, and numerous other expenses that must be paid.

Percentage Refunded
1 week ....80% refunded
1 to 2 ....60%
2 to 3 ....40%
3 to 4 ....20%
4+ 0%

University Retention

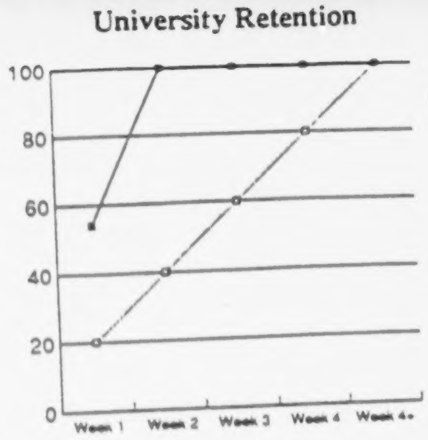


University Retention



Quadratic Model – This model is not practical in relation to the University's refund schedule. While the initial rates run slightly below current values, from there they increase dramatically. This proposal would undoubtedly reduce the number of students enrolled each semester. The rate, initially starting at 80%, would decrease to 20% the second week before reaching 0% at the third week. The rapid increase in rates would hurt Clemson's financial status in the long run. Any decrease in enrollment will further the economic problems and hurt the University's ability to provide a quality education.

Percentage Refunded
1 week ....80% refunded
1 to 2 ....20%
2 to 3 ....0%

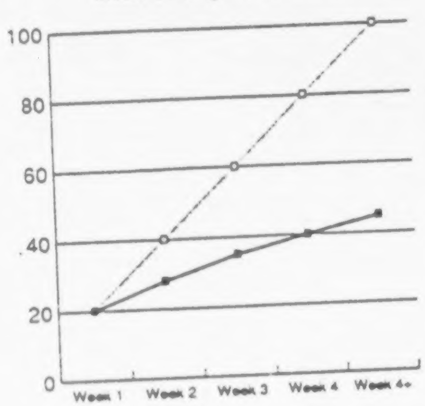


Exponential Model – This model poses an even more unrealistic approach than the quadratic model. The rate starts higher and increases faster than earlier examples. Although the exponential model allows for a total refund before the course ever begins, the subsequent increase is so drastic, it would only encourage student protest and cause a decrease in enrollment.

Percentage Refunded

1 week	46% refunded
1 to 2	0%

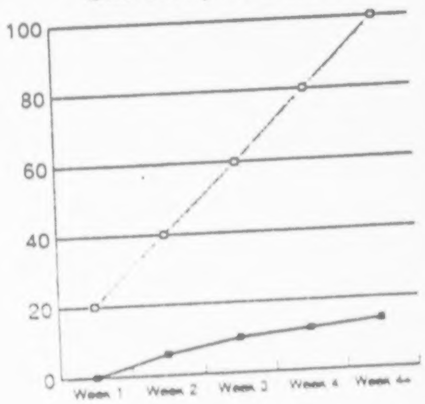
Square Root Model – While this model proves to be more promising than either the exponential or quadratic models, it still would not solve the economic dilemna of the school. The rate of increase is too small to support the budget of the University and the initial percentage is insufficient, as well. This model, while provideding a more moderate increase, is encouraging to students.



Percentage Refunded

1 week	80% refunded
1 to 2	72%
2 to 3	65%
3 to 4	60%
4+	55%

Logarithmic Model – Initially, this model does not cause any decrease in the refund until the second week. From there the rate slowly progresses and a minimal slope is in effect. With this model, students would receive the same refund amount after the fifth week that they currently receive after the first week of enrollment. This plan, while appealling to students, would not be economically sound for the University causing course cutbacks.

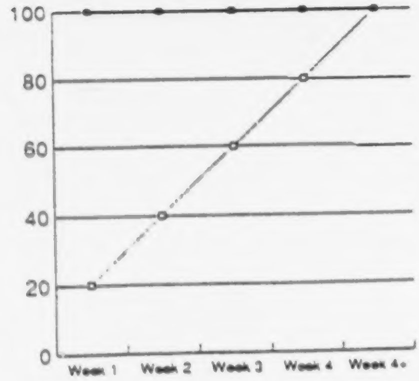


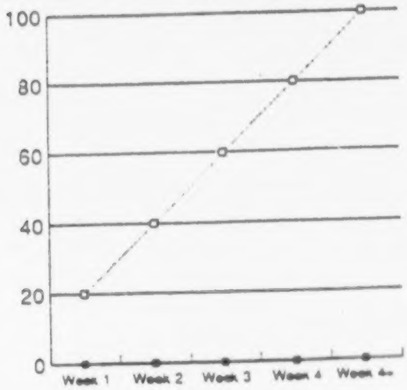
Percentage Refunded

1 week	100% refunded
1 to 2	94%
2 to 3	90%
3 to 4	88%
4+	86%

No Refund Model – Impractical. This model would temporarily remedy the economic problems of Clemson but student opinion would quickly result in a drastic reduction in enrollment, causing disaster for the school. There are few, if any, benefits for students with this plan.

Percentage Refunded
1 to 4+ ....0% refunded


University Retention

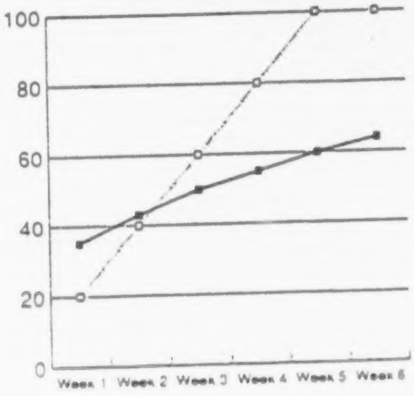

University Retention

Complete Refund Model – This model would definitely have the needed student support. But, it would quickly ruin any attempt by Clemson to overcome budget cuts and reestablish lost classes, also, there would be no compensation for the money already spent on paperwork and time.For students, this plan would provide more flexibility in finding necessary classes.

Percentage Refunded
1 to 4+ ....100% refunded

Suggested Proposal:

Square Root Model + 15% Non-Refund – This could serve as an acceptable alternative to the present schedule. By imposing a 15% base, the University covers any costs incurred while registering and recording student information. At the same time the length of the withdrawal period is lengthened to six weeks instead of four allowing the student to become familiar with the course. However, each student who withdrawals after the first four weeks is assigned extra withdrawal hours; but, no economic penalties are enforced.The decrease in the refund rate schedule encourages students to participate longer and saves them anywhere from 10% to 36% after the first three weeks.


University Retention

Percentage Refunded
15% non-refundable
1 week 65%
1 to 2 ....57%
2 to 3 ....50%
3 to 4 ....45%
4 to 5 ....40%
5 to 6 ....36%

*Inaccurate records prevented exact predictions to be made. All examples are based on current estimates.

MTSCH 102 PROJECT I

Group 7
Amy Melto
James Stu
Jessica Wa

We as the members of the Anderson Consulting Company have thoroughly reviewed the current fee refund schedule of Clemson University and have concluded that an alternative plan would be more beneficial.

The company agrees with Clemson's current plan of "No refunds will be made on a semester's tuition and fees after four weeks from the last day to register." The student should also send a request to the Office of Business Affairs prior to the beginning of the next fall/spring semester or subsequent summer term to be eligible for a refund. However, the existing percent refunded between weeks is not the best choice for all parities involved.

The University would initially attract more students if they were promised a refund of more money after the first week of enrollment. The University would gain more money in the long run if the students had the assurance that they would be refunded more than 80% after the first week. More students would enroll than would drop out which would allow for an increase in funds for Clemson. We agree with the linear model because the refunds should be in equal increments. If the students were offered a larger percent of the entire tuition during the first week they would be more likely to make a premature decision to withdraw. If they were refunded at equal intervals each week then they would be more likely to stay at the University for a longer period. This would be more beneficial to the school because they would refund less to students.

As the group convened we looked at all alternatives and decided that the linear model was the best choice.

quadratic

The plan that we studied in the quadratic is as follows:

1 week or less	90%
2 weeks or less	75%
3 weeks or less	55%
4 weeks or less	30%
after week 4	0%

After reviewing this plan, we have decided that this would only benefit the student. The refunds are entirely too high and there would be money lost by the university. The university would only receive 45% of the tuition after the third week of enrollment. The time and money provided by the university will never be compensated for if this plan were installed. The only members of the commission that would benefit from this plan are the parents of the students who withdraw. Everyone else will be effected by the increase of state taxes made to support the school. The farmers, businessmen, and plant managers will feel the tax increases as well as their employees. The politicians will be pressured by different groups to deal with the tax increase. The educators will be effected because the quality of education will decrease at Clemson University. There will be fewer programs and resources.

no refund 0%
after enrollment

The no refund plan would be more than fair to the school but less than fair to the students and parents. Some businesses provide tuition for their employees who are enrolled in classes. The business would loose all of their money if the student had to withdraw. The business sector would refuse to pay for any tuition for their employees to further their education. There would be less growth in companies. This plan would be very unfair to all parties involved as well as the university. Students would less likely enroll knowing that they would not receive any percent of their tuition if they were to withdraw.

complete refund 100%
4 weeks or less

The complete refund plan would not be profitable to the university at all. The student could take advantage of the resources and programs offered at the university for up to four weeks without any financial investment. The instructors and faculty would have undergone a lot of preparation and spent a lot of money. The university would loose tremendously.

exponential

The exponential plan that we studied is as follows:

week 1	90%
week 2	84%
week 3	72%
week 4	48%
after week 4	0%

As you can see by this model, the only party to benefit would be the student. The school would only receive 52% of the total tuition by the fourth week. The results would be very similar as the previous quadratic model but they would be even more detrimental to the university. Needless to say our company quickly ruled out this option.

square root

The square root plan that we studied is as follows:

week 1	90%
week 2	88.27%
week 3	85.37%
week 4	76.60%
after week 4	0%

This plan was found to be outrageously expensive to the parties involved and a poor choice for Clemson. The percent refunded is much too high and all of the negative effects of the previous models will certainly apply.

logarithmetic

The logarithmetic model that we studied is as follows:

week 1	90%
week 2	73%
week 3	52%
week 4	24%
after week 4	0%

Once again our company has found fault in another model for fee returns. This model will benefit the students more so than the other parties involved. After three weeks, the student will have received more than half of their tuition back. The university has invested a great deal of time and energy into setting up the courses and should be given more compensation than the logarithmic model allows.

The Anderson Consulting Company has rightfully chosen the linear model for Clemson's improved fee refund schedule. Our company is confident that an increase of ten percent in the refund for the first week will be very beneficial to all members of the Higher Education Commission.

The linear plan that we have selected is the following:

refunds for one week or less	90%
refunds for less than 2 weeks	67.5%
refunds for less than 3 weeks	45%
refunds for less than 4 weeks	22.5%
refunds for more than 4 weeks	0%

This plan will allow the student to be refunded 90% of the initial tuition during the first week. It also diminishes in equal increments. The university as well as the students will benefit form this model. The public debate will come to a peaceful settlement if this plan were to be put into action.

GREENVILLE NEWS

Clemson University officials finally acknowledged the rioting students and changed the refund system to curb the large number of students dropping out of the university. The students complained that the university was taking too much money from them when the classes they needed were unavailable. A sub committee to review the refund schedule was hired by the university to see what changes could be made. This committee found that 90% of the money should be returned to the students during the first week instead if the 80% the university has been returning. The ten percent difference would be enough to cover any processing fees charged by the school. After the first week, the refund drops 22.5% where in four weeks the student will get no refund. The old refund schedule gave the students a 20% drop every week when starting at 80%. There has been plenty of positive feedback for Clemson's new plan and it looks like it is going to be a great success.

Group 7

Meeting #1
Thursday at 2:30
Library

Today we analyzed each option and chose the one that we preferred. We worked out percentages and equations to find numbers that fit each model. We decided that whichever model we chose the student would receive 90% after the 1st week of enrollment. We went through each model and decided that the equal increments of the linear model would be the best suited for the Students of Clemson.

We rejected every other model because it gave more $ to the Student.

Each member of the group helped to work on the percentages and #s for each model. We divided the responses and made a meeting time for Monday at the same time and location

Group 7

Meeting #2
Monday 2:30
Library

Today we exchanged papers and ideas we talked more about how we wanted to organize the response and present our proposal we only met for 20 min because a lot of work was completed at home for each member.
We divided more duties and the meeting was ajourned

Final Project: Summer Session 1993

Student Samples

Three Samples Given (written project and summary of presentation):

- Group 4
- Group 7
- Group 10

Memorandum

To: all employees of First Trustworthy Bank

From: Goude, Heckel and Wechselberger, hired consultants of First Trustworthy Bank

Date: effective July 30, 1993

Re: new employee policy concerning CD rate inquiries

Recently, First Trustworthy BAnk has received frequent questions from customers concerning CD rates. The simplest and most reasonable method to explain "years to double" is the "rule of 72".

For those unfamiliar with the "rule of 72", the rule easily calculates the number of years an investment takes to double in value. By dividing 72 by the appropriate interest rate, the rule solves the question. For example, if the interest rate is six percent, 72/6=12. Therefore, an investment with a six percent interest rate will double in twelve years, according to the "rule of 72".

However, this number is only a close approximation and not necessarily the actual doubling time for any investment. For rates higher than eight percent the "rule of 72" becomes more and more unreliable since the rule indicates a time preceding the actual doubling time. Thus, it would be inappropriate to use the rule for an interest rate higher than eight percent without informing the customer that the time is indeed an estimate.

The bank and its employees can alleviate this problem by providing a three month grace period on investments with interest rates higher than eight percent. This grace period would cover interest rates up to and including twenty percent.

Therefore, "when you are asked" about current CD rates, apply the "rule of 72" and inform the customer of the bank's three month grace period on the doubling estimate of investments with interest rates higher than eight percent. Failure to do so might create problems of customers holding the bank liable for misrepresentation.

Caveat statement (to be repeated after approximate time is given)-
The bank wants all its customers to be informed of the fact that the time period given for the doubling of CD values is an estimate, and a three month grace period should be added to the period to account for any error in the calculation of the time period.

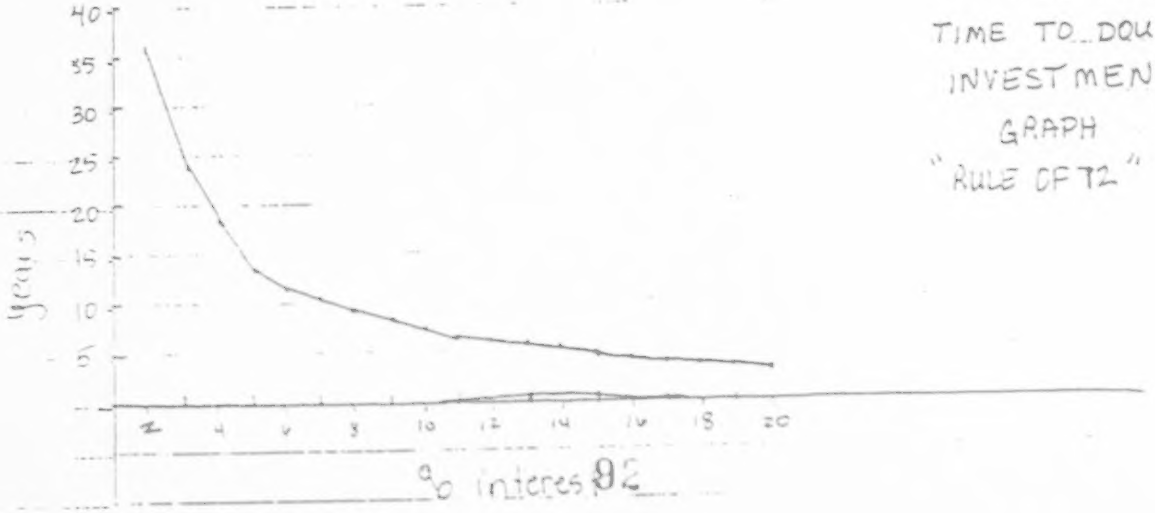
VALUES FROM THE "RULE OF 72"

EQUATION:

$$\frac{72}{P}$$

MONTH ≈ .0833333 year

%	TIME TO DOUBLE	TIME years months
2	36	36 y
3	24	24 y
4	18	18 y
5	14.4	14 y 6 mo
6	12	12 y
7	10.2857	10 y 4 mo
8	9	9 y
9	8	8 y
10	7.2	7 y 3 mo
11	6.54	6 y 7 mo
12	6	6 y
13	5.53	5 y 7 mo
14	5.142	5 y 2 mo
15	4.8	4 y 10 mo
16	4.5	4 y 7 mo
17	4.23	4 y 3 mo
18	4	4 y
19	3.78	3 y 10 mo
20	3.6	3 y 8 mo



TIME TO DOUBLE INVESTMENT GRAPH "RULE OF 72"

ACTUAL DOUBLING TIME

"Rule of 72" from previous graph|z

EQUATION:
$\left(1+\frac{p}{100}\right)^{N}$

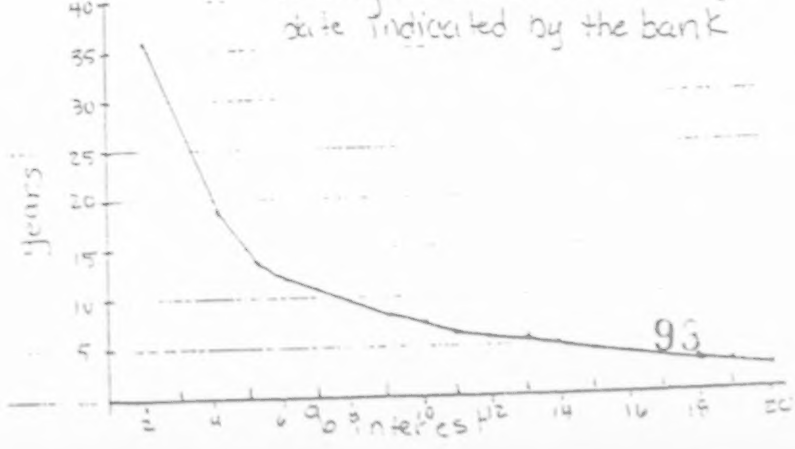
MONTH ∝
.0833333
year

%	TIME TO DOUBLE (years)	TIME years months	TIME years months	DIFFERENCE BETW RULE OF 72 & ACTU
2	35.002	35y 1mo	36y	11 mo. ★
3	23.44	23y 6mo	24y	6 mo ★
4	17.67	17y 9mo	18y	3mo ★
5	14.20	14y 3mo	14y 6mo	3mo ★
6	11.89	11y 11mo	12y	1mo ★
7	10.24	10y 3mo	10y 4mo	1mo ★
8	9	9y	9y	—
9	8.04	8y 1mo	8y	1 mo
10	7.27	7y 4mo	7y 3mo	1 mo
11	6.64	6y 8mo	6y 7mo	1 mo
12	6.116	6y 2mo	6y	2 mo
13	5.671	5y 9mo	5y 7mo	2 mo
14	5.290	5y 4mo	5y 2mo	2 mo
15	4.959	4y 12mo*	4y 10mo	2 mo
16	4.670	4y 9mo	4y 7mo	2 mo
17	4.414	4y 5mo	4y 3mo	2 mo
18	4.187	4y 3mo	4y	3 mo
19	3.984	3y 12mo*	3y 10mo	2 mo
20	3.801	3y 10mo	3y 8mo	2 mo

*slightly less than entire 12 months, or year

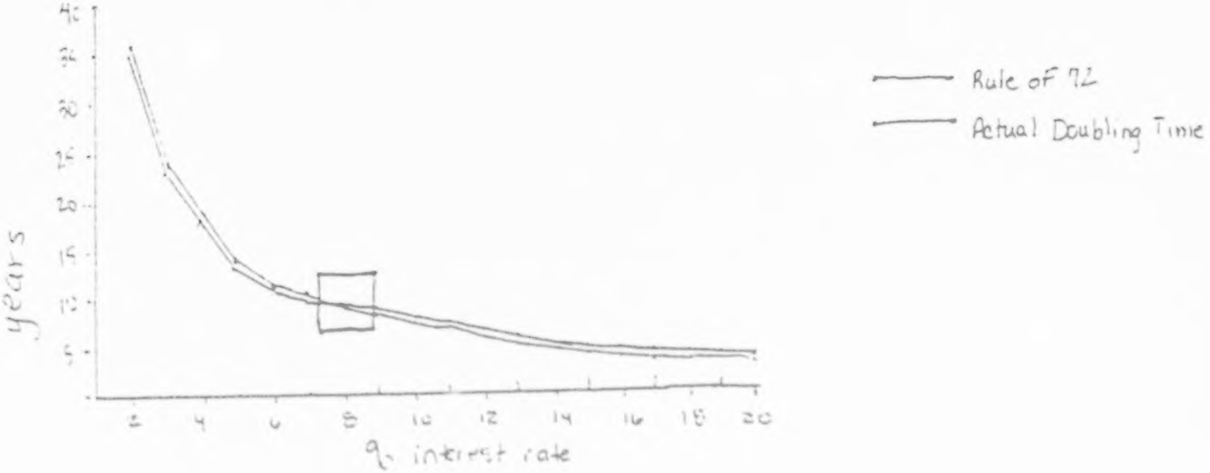
* times in red indicate discrep between "Rule of 72" and actua time whereby the bank might be in danger of liability to the customer

★ indicates customer's investment would actually double before maturity date indicated by the bank



ACTUAL TIME TO DOUBLE INVESTMENT GRAPH

COMPARISON OF GRAPHS



PROBLEM: A comparison between the graphs of the two lines shows that at approxiamately 8% interest, the lines intersect. Therefore, the actual doubling time is greater than the time given by using the "Rule of 72."

SOLUTION: Referring back to the tables, the discrepancy of time between two (2) and twenty (20) percent does not exceed three months. In order to accomodate for the difference in time, a solution must be reached to combat any liability on the part of the bank. Since the "Rule of 72" is simpler for tellers to calculate in a short amount of time, it can be used provided that the customer is informed that a three month grace period will be allowed for investments accruing eight (8) percent interest or more

A Practical Approach-

A customer might be inclined to ask, "How does an interest rate percentage affect me in terms of dollars?" You, as a worthy public service provider, can inform the customer as to the numerous benefits associated with varying interest rates. For example, bank researchers have discovered that for the last four years, interest rates have been fluctuating at a constant rate yearly. For instance, CD rates held steady at 5.3% during the summer months of May-August during the years 1988-92. Also, the interest rates increased to 6.8% and remained during the winter months of November-February of the same years. Therefore, it would clearly be in an investor's best interest to purchase CD's during these winter months. You can inform the customer that, by using the "rule of 72", it would take approximately 2.85 years less to double his/her investment with a 6.8% interest rate than with a 5.3% rate. Situations such as this can be beneficial to both the customer and the bank.

Final Project Presentation Summary: Summer Session '93

Group 4

Presentation:

This group began by setting up the overhead panel that would allow them to display the screen of their Sharp calculator. They used the calculator to graph the actual values for doubling time and the estimated values given by the Rule of 72. They indicated that both graphs were very similar and close together. They then used transparencies they had prepared to state the problem and their solution. The statement of the problem was not clear. Next they displayed a chart of values for both the actual time to double and the Rule of 72 approximations. They calculated the differences between the two values and looked at how small the differences were. Finally, they used a "what if" strategy by contriving fictional data for the years 1988-1992 at several different interest rates. They graphed this data with overlaid graphs which was very hard to read. All three students participated in the explanation and presentation of findings.

During their presentation, they did emphasize that the Rule of 72 was an approximation. They said that between 0% and 8%, there was no problem since the money actually doubled before the estimated time. However between 8% and 20%, the Rule of 72 underestimated. This is a problem. The solution would be to put in the caveat a three month "grace period." These students had a far better grasp of the problem than most. They located trouble spots and then came up with ways to deal with them.

Q & A:

The first question was about the last graph. The grader wanted to know if all the interest rates were real. The student emphasized that they made up this data to show how and where the Rule of 72 could be used. Next the grader stated that he did not understand the problem stated on the first transparency. The student tried to explain what they meant, but kept stumbling over his words. The grader kept after him. Finally the student ended up covering up the transparency and stated the problem verbally. He said that for 8% to 20%, the bank may be held liable for misrepresentation since the money does not actually double by the estimated time given by the Rule of 72.

The grader went back to the last graph and asked the student to explain it to him again. He followed this with, "Show me May, 5.53% for 1991." The student gave a rough estimation of where this would be on the graph. He then said that the group looked at seasonal fluctuations of rates--higher in the winter--lower in the summer. They could use this to advise customers when to invest their money.

Finally the grader asked if the group would tell a customer that their estimate was accurate to within three months or would they just add three months to the Rule of 72 estimate. The student agreed that they would just add the three months to the estimated doubling time.

P

MTHSC 102
Project III
Group 7

James Stukes
Amy Melton
Jessie Warren

FIRST TRUSTWORTHY BANK

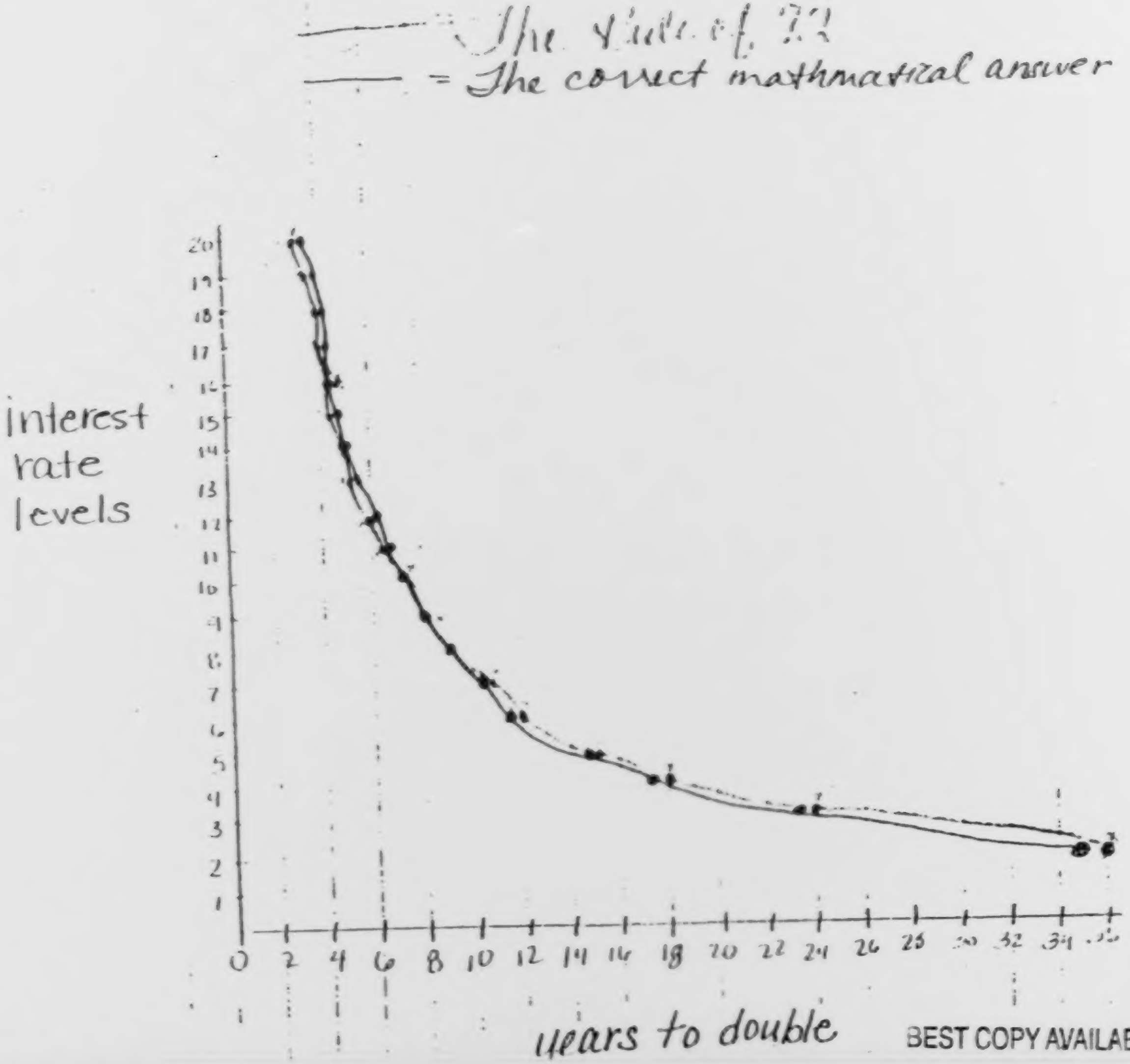
When you are asked......

A businesswoman approaches a teller and wishes to buy a Certificate of Deposit. The CD that she is purchasing has an interest rate of 5%. The teller will inform her that the CD will double in value in 4.4 years using the "rule of 72". This is an approximation of the time it will take the CD double.

The "rule of 72" is a means of calculating the number of years that it takes money to double. The rule is calculated by dividing 72 by the annual interest rate expressed as a percent.

When explaining to customers "years to double" comparisons it is very important to clearly demonstrate that the rule does have some error. For example, if we divide 72 into 20, the answer is 3.6 years. We must explain to the customer that it will take 3 and 1/2 years for the money to double. We cannot dismiss the numbers after the decimal point. It is important for legal reasons for each teller to explain that the "rule of 72" is only an approximation. The customer must understand that the money must be invested for a certain number of years at a certain percent rate in order for the money to double in the manner in which the "rule of 72" approximates.

ATTENTION TELLERS:
You must explain to the customer that the "Rule of 72" is an approximation and not always accurate. This will prevent the bank from being involved in a legal suit.


The Rule of 72
= The correct mathmatical answer
interest rate levels
years to double
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36

First Trustworthy Bank
[redacted]

When you are asked...

"The Rule of 72" is a means of calculating the # of years that it takes money to double. The rule is calculated by dividing 72 by the annual interest rate expressed as a percent.

When explaining to customers "years to double" comparisons it is very important to clearly demonstrate that the rule does have some error. For example, if we divide 72/20, the answer is 3.6 years. We must explain to the customer that it will take 3 1/2 yrs. for the money to double. We cannot dismiss the numbers after the decimal point. It is important for legal reasons, for each teller to explain that the rule of 72 is only an approximation. The customer must understand that the money must be invested for ~~the~~ a certain # of years at a certain % rate in order for the money to double in the manner in which the rule of 72 approximates.

Reminder: ~~Be sure to~~ It is a must to explain to the customer to be informed that the "Rule of 72" is an approximation. If the customer is not informed the bank could be involved in a legal suit.

To the first Trustworthy Bank Tellers

A Businessman approaches a teller and wishes to buy a CD. The CD he is purchasing is at 5%. The teller must tell him that the CD will double in value in 14.4 years using the "rule of 72." This is an approximation of the time it will take the ~~to~~ CD to double. When you are asked how many years it takes for a 5% CD to double the teller must say 14.4 years. But in actuality if you use the $(1+P)^n$ it will double in 14.2 years which is faster than the client expected. It will be NECESSARY to Explain to customers if the money will not double in the time the Rule of 72 claims it will double in.

BEST COPY AVAILABLE

Program 3

$2=(1+P)^N$

P= 2
N= .6309_9753

P = 3
N = .5

P= 4
N= .430676558

P= 5
N= .386852807

P= 6
N= .356207187

P= 7
N= $.\overline{33}$

P= 8
N= .315464876

P= 9
N= .301029995

P= 10
N= .289064826

P= 11
N= .278942945

P= 12
N = .270238154

P= 13
N = .262649535

P= 14
N= .255958024

P= 15
N= .25

P= 16
N= .244650542

P= 17
N= .239812466

P= 18
N= .235408913

P= 19
N= .231378213

P= 20
N= .227670246

$$N = \frac{1.92}{3\left(1-\frac{D}{100}\right)}$$

2 = 35.00
3 = 23.44
4 = 17.67
5 = 14.20
6 = 11.89
7 = 10.24
8 = 9.00
9 = 8.04
10 = 7.27
11 = 6.64
12 = 6.11
13 = 5.67
14 = 5.29
15 = 4.95
16 = 4.67
17 = 4.41
18 = 4.18
19 = 3.98
20 = 3.80

BEST COPY AVAILABLE

%	grs ~~is~~ double
~~30~~ 20	3.6
19	3.78
18	4
17	4.23
16	4.5
15	4.8
14	5.14
13	5.53
12	6
11	6.5
10	7.2
9	8
8	9
7	10.25
6	12
5	14.4
4	18
3	24
2	36

Final Project Presentation Summary: Summer Session '93

Group 7

Presentation:

This group had very little to offer in terms of a presentation. The only visual aid that they used was the calculator. They used this to display scatter plots of the Rule of 72 values. From this they went on to explain that the rule of 72 was an approximation and that bank customers should be aware of the discrepancies between the approximate values and the actual times to double. All three students participated in the presentation, the majority of which was read from note cards. They never stated what the Rule of 72 was or explained how they found the actual doubling times.

Q & A:

The grader did not ask many questions. I am not sure why this was the case. However, when asked the students were able to successfully explain that the Rule of 72 overestimated after 8% and underestimated before 8%.

FIRST TRUST WORTHY BANK

MEMORANDUM
TO: Bank employees
FROM: Triad consultants Group # 10

Lou Ann Waldrop, Data Recordist
Robert Roberts, Reality Specialist
Eliza Roberts, Organizing Manager

RE: An overview of the First Trustworthy policy on the Rule of 72 with comparison to the real formula for years-to-double C.D. investment plans.

As some of you may already know, the First Trustworthy Bank uses the Rule of 72 plan to predict at what percentages of interest and over what period of investment time our investors can plan to double their investment. Because it is a mathematical model, it may not always be accurate in the actual world of finance. The errors which might occur, if not mentioned to the customer, could result in a liability beyond the best interests of our institution.

It is for this reason that we feel it is essential for you to be aware of the particulars of the model. Attached to this memo, you will find a table of values which include the projected years to double using both plans. You should note the citing of error between the two plans and should be able to explain this to your customers. A graph has been included which shows how closely the two plans relate, however the area of greatest concern has been indicated with regard to the increase in error in one particular area of the Rule of 72.

A training and testing day will be set up at a later date to insure that you understand and can utilize this information. Should you have additional questions, please feel free to contact any of the following people who will be glad to assist you:

Dr. Don Latorre, CEO
Dr. Cindy Harris, President
Dr. John Kenelly, Information's Specialist

Thank you for your attention to this important issue concerning our working policies. Please remember that it is essential that we provide our customers with as accurate information as possible.

THE RULE OF 72

The Rule of 72 is a mathematical model which tells us that the number of years "N" to double the investment is equal to 72 divided by the percent interest "P." The formula looks like this:

$$N = \frac{72}{P}$$

A table of values is provided for you on the next page for easy reference.

THE REAL VALUE

The Real number of years that it takes to double an investment is a mathematical model as well. It projects that the number of years N to double is equal to the natural log of 2 divided by the natural log of the combined sum of 1 plus the percent interest divided by 100. The formula looks like this:

$$N = \frac{\ln 2}{\ln(1 + p/100)}$$

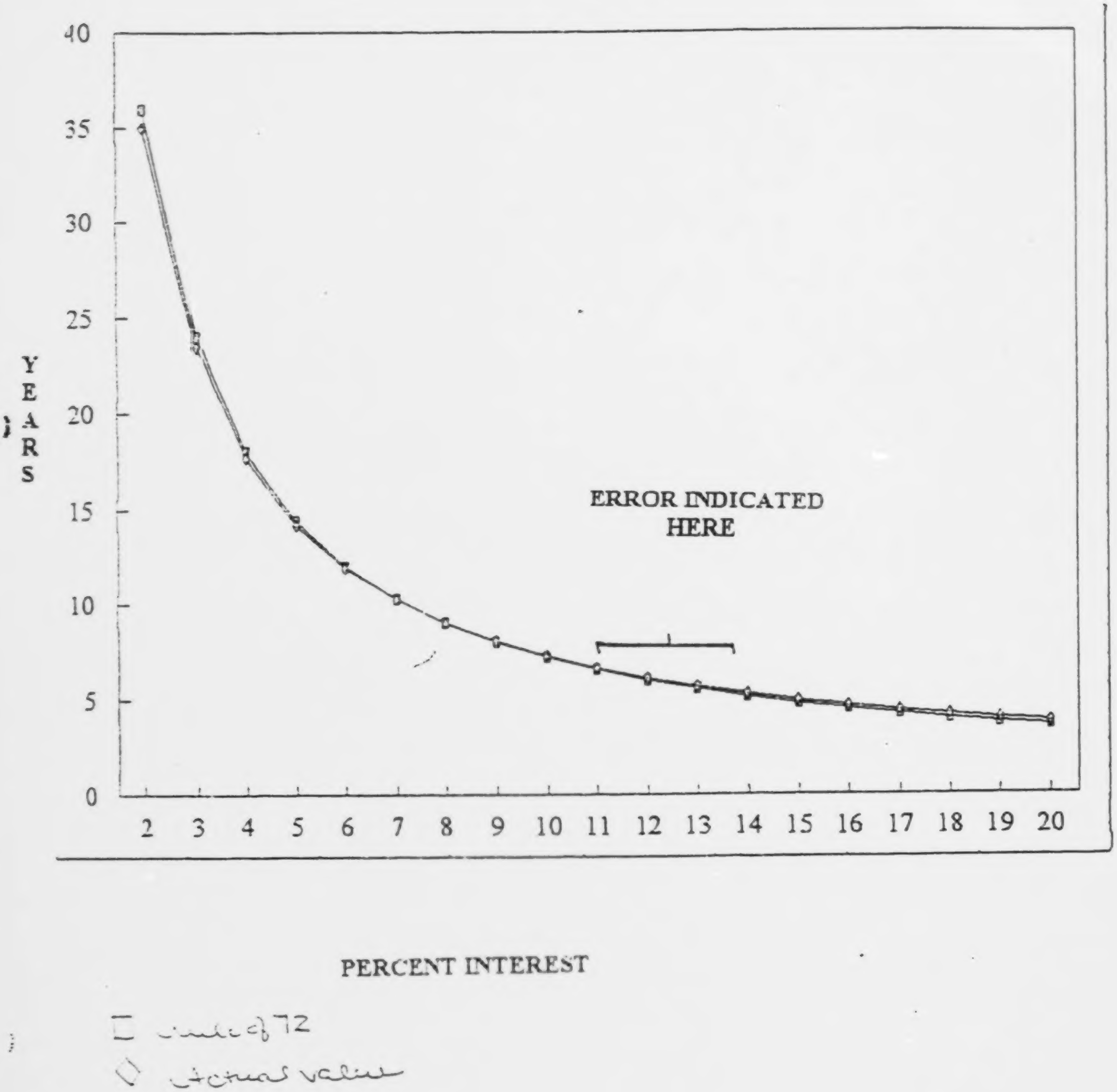
Please refer to the tables to compare the Rule of 72 with the Real model. You will note that the two have very similar values. Note the "slope of the tangents" columns for both models. These values act as a speedometer to show us the increasing or decreasing rates of change. The amount that these values differ between the two models may indicate errors for which the bank may be held liable. In this case, the number of years to double continues to decrease with increasing interest rate. The area in question occurs between 11 percent, 12 percent, 13 percent and 14 percent interest. The number of years to double has been decreasing at a decreasing rate of change. The error is -0.03 at 11 percent. It ~~[illegible]~~ increases to -0.01 at 12 percent, and then ~~in~~decreases to -0.02 at 13 and 14 percents. This change between 12 and 13 percent is evidence of error to be particularly aware of. If you refer to the graph included, you wil see again the similarities of the two models, and you will also see the tangent lines at 12 percent which indicate that the rates become less similar here.

TABLE

FIRST TRUSTWORTHY CD COMPARISON

	YEARS-TO-DOUBLE		SLOPE OF TANGENTS		(LIABILITY
Percent interest	Rule of 72	Real plan	72's y'	Real's y'	Diff. or erro
2	36	35	-17.9	-17.3	-0.6
3	24	23.45	-7.9	-7.7	-0.2
4	18	17.67	-4.5	-4.3	-0.2
5	14.4	14.21	-2.9	-2.8	-0.1
6	12	11.9	-2	-1.9	-0.1
7	10.29	10.25	-1.5	-1.4	-0.1
8	9	9.01	-0.89	-1	-0.1
9	8	8.04	-0.89	-0.86	-0.03
10	7.2	7.27	-0.72	-0.69	-0.03
11	6.55	6.64	-0.6	-0.57	-0.03
12	6	6.12	-0.49	-0.48	-0.01
13	5.54	5.67	-0.42	-0.4	**-0.02**
14	5.14	5.29	-0.37	-0.35	**-0.02**
15	4.8	4.96	-0.32	-0.31	-0.01
16	4.5	4.67	-0.28	-0.27	-0.01
17	4.24	4.42	-0.25	-0.24	-0.01
18	4	4.19	-0.22	-0.21	-0.01
19	3.79	3.99	-0.2	-0.19	-0.01
20	3.6	3.8	-0.18	-0.17	-0.01

GRAPH OF RULE OF 72 MODEL AND REAL MODEL



Final Project Presentation Summary: Summer Session '93

Group 10

Presentation:

This group started with a transparency that showed a memo explaining the Rule of 72 and actual doubling time formula. They then proceeded to use a transparency that showed the overlaid graphs of these two functions and compared them. Also they used the calculator to show the actual graphs overlaid. From there they presented a data table transparency that showed the interest rates, doubling times and something labeled "y'" that the students called "slopes". In regards to these "y'"'s, one student drew sketches of the two graphs on the board. She then proceeded to choose the points on each graph corresponding to a specific interest rate. She sketched the tangent lines to these points. She said that the separation between the tangent lines indicated that the errors between the approximate values and the actual values were increasing. This group was one in a small minority that discussed "slopes" and looked at tangent lines.

Although all three students participated in the presentation, there was one student that basically spouted garbage the entire time he talked. It was evident that he did not know what he was talking about. One of the other students said that both the Rule of 72 and actual doubling time formula were models--each was a different approach to the problem. She didn't seem to be real clear about what she meant by "approach" and which "problem" she was referring to.

Q & A:

The grader began by asking what the y'-s were telling them. The first student fumbled around by talking about "integer values" for the graph. Then another student jumped in by saying that she thought the grader might want to know how they got y'. She explained that y' indicated slope = rise over run. Then she said that it was going to take longer for money to double at lower rates. The third student interjected that the percent was decreasing at each year. These last two statements were not challenged by the grader for further explanation. They were then asked why is necessary to look at y' values. They said that using actual differences was okay, but they needed slopes to give a more definitive answer. Again this was not challenged

It seemed to me that from the discussion about slopes that the students had some sort of superficial understanding of what they were talking about. However, for the most part, they seemed to be throwing around alot of words because they thought it would be expected of them.

Student Interview Summaries

Summer Session 1993

Five Student Interview Summaries:

- **July 26, 1993**
- **July 28, 1993**
- **July 29, 1993**
- **August 2, 1993**
- **August 3, 1993**

Student Interview: July 26, 1993

Sex: Male
Race: African American
Age: 20
Year: Sophomore
Major: Management
Number of times in MTHSC 102: 1
Other Math Courses at Clemson: MTHSC 104

I asked the student about his High school mathematics experience and he said that he really enjoyed it. He felt that it was one of his strongest subjects. Yet, he felt that math was just a bunch of numbers and that all he was required to do was "do the test". He told me that his attitude towards math has changed since this class began. He thinks that math is neat. He really enjoys the projects. They hit close to home. He can relate to them. He said that he felt he had a better sense of buying things and how things worked.

He really enjoyed the group activities. He realized that there would always be conficts, but in the group, each person would present their views on a problem and then the group would try to work towards a consensus. He said that many times people would bring up points that he had not thought of. He felt that one of the greatest aspects of group work was that he had to clearly explain what he meant to the others in the group. This required him to know the subject much better and to think out what he had to say.

When asked what he didn't like about the course, he replied thta there were no weak points. He liked everything. He said that this course made math easier to understand. He then went on to discuss the testing situations he had experienced. The tests required you to know the basic stuff to work problems. He had to know how to work the calculator. But there was nothing too taxing on the exams. As far as the calculator is concerned, he said he had fun with it. At the beginning, when he was handed the calculator, he considered dropping the class. He was sort of scared of the calculator. The class looked really tough. But he decided to hang in there and has really enjoyed himself. The calculator has made him think what his future holds in management as far as technology is concerned. He definitely sees himself using some of the skills he has learned this summer in his career.

I asked him to objectively evaluate his progress this summer. He said he had made progress every week. As the course progressed he began to feel more comfortable with the material. He began asking and answering questions. He really liked the fact that there was no right answer. This made him feel freer to answer questions. He felt that he could contribute something to the discussion because he felt confortable with the subjects discussed. Along this line I asked him about his previous class at Clemson. He really hated it. He said that if he didn't understand anything that he usually didn't ask many questions. The reason was because when the teacher answered him, they flew by so fast he didn't get it. This was the type of attitude he brought into this course at the beginning. He said he was quiet at first because he felt it would be just like the other class -- useless to ask questions because he either wouldn't get an answer or the answer would be given too fast. In this course, he soon discovered that since the problems were related to real life situations he could understand more of what was going on mathematically. When he asked questions, the answers made sense. They weren't just a bunch of numbers and formulas.

I asked him to sum up his experience. He said that this class was like the real world. The things he learned in it are relevant to his life. He said it was a "great class" and that he thinks it should be a "required class." I asked him why he thought this. He said because it teaches you real life situations. He referred to the car buying situation and stock market curves. He said he felt like he was better prepared to deal with real life problems that are going to come up in the future. This was one of his better classes.

Student Interview: July 28, 1993

Sex: Male
Race: White
Age: 20
Year: Sophomore
Major:
Number of times in MTHSC 102: 1
Other Math Courses at Clemson: MTHSC 104 (twice)
Couldn't grasp it. Computers bothered him - "computer illiterate"
No partial credit.

We started off by talking about his high school math experience. He went to a very small school. The teacher was readily available and took time to explain things to him. He took physics and advanced math in high school. When he got into 104, he asked questions but found that teacher went over it too fast. He could not get it. He didn't like it at all. The second time around he was pretty familiar with how things worked. He memorized enough to get by. From this his definition of math was just a bunch of numbers. You would never use it. After this class, he has a different attitude towards math. He has actually seen where it can be used in the world. He could see using models in presentations. The Walmart example seemed particularly interesting to him.

He really enjoyed the class. He liked the fact that it was small. The teacher was very personable. He also liked the fact that the projects were "real worlding it". He also seemed to enjoy working in groups. He said that there were differences of opinions. But it was nice because it was almost guaranteed that one of the three knew what was going on. They ended up splitting apart the problems and working on them individually.

As for the calculator, he enjoyed it. He said that graphs made more sense now. On the first day, he was really excited by getting the calculator. He went home and played with it.

This student hopes to take 207 in the fall. He feels that his attendance was not too bad. (He had missed 3 times at this point.) He felt that he had done his share in the projects and on the tests. The last test he totally blanked out on. He said that he felt that he learned slower than most people. When asked to grade himself, he gave himself a B.

Student Interview: July 29, 1993

Sex: Female
Race: White
Age: 23
Year: Fifth Year Senior
Major: English
Number of times in MTHSC 102: 3 (failed once, dropped once)
Other Math Courses at Clemson: MTHSC 101 -stats
MTHSC 105 - dropped

This student has a history of trouble with math. She took Algebra I and II and Geometry in high school. She made D's in all of them. She really hated math. It confused her. She had an extreme case of math phobia. She had physical reactions such as getting hot and sick. When she got to college it got worse. When asked to give a definition of mathematics, she said it was "hell". She always hated to go to class. In order to survive, she took a Tylenol before and after class. She said math was always "this way or no way". That is the reason she likes English so much. You can use imagination and emotions there. In math classes, this student felt slower than everyone else. She asked questions but got so far behind.

Her attitude has changed somewhat since this summer. The class was a definite improvement. She saw where mathematics was applied to real life. She liked the calculator. It had systematic step she could memorize and follow. That gave her some comfort. This wasn't always the case. The first day when she got the calculator she thought, "I'm dead." She was scared of it. The fact that this class was structured so that algebra was de-emphasized made her feel better.

There were some problems faced in this course. One source of trouble came from the group projects. She was placed in a group with an extraordinary intelligent student. He dominated the groups proceedings. He became very frustrated with this student. He would not listen to the other group members. He did not want them to touch his project. The student I interviewed would have her other group member explain what was going on. However, that member did not always understand what was happening. Pretty soon they just decided to be quiet and let the dominating student control. The interviewed student said that hardly anything on the projects they turned in belonged to her. She felt very worried about the third project. She had no idea what was going on there. She said she honestly didn't know if she learned anything form that project. She felt as if she would get to a point of some understanding and could go no further. She said she was worried about the presentation. She felt like all she would be able to do would be handle the overheads.

Tests were also problematic. It was pointed out by this student how different the tests were from the projects. She passed both of the tests. She did say that to stop having tests would be a big mistake. Students would have no motivation. She said that she read through the first book and no further because she knew she would not be tested on it.

Finally I should mention her standing on the pass/fail grading situation. I decided to interview this student the day she received her third "pass" for the course. This automatically guaranteed her passing of the class. Since this was her third attempt and having a job offer hanging in the balance, she was very excited. However, she mentioned that the pass/fail idea was not motivating to the student. Once he/she got three passes, they didn't have to come to class. anymore.

Student Interview: August 2, 1993

Sex: Male
Race: White
Age: 22
Year: Graduate Student (graduated from the Citadel with degree in Business Administration)
Major: MBA
Number of times in MTHSC 102: 1
Other Math Courses at Clemson: (at Citadel)
College Algebra, Calculus

This student has a rather rich mathematics background. In high school he took everything they offered. He had Algebra I & II, Geometry, Trigonometry, Pre- Calculus, and Calculus. The reason he did so was because at that time he planned to be an Air Force pilot. After failing a few courses, he saw that he would not be able to fulfill this dream. So he began to look at business. He saw that he was pretty good at mathematics "when I tried to do it". After realizing he would not make it as a pilot, he lost interest in his mathematics courses. He had no motivation. His grades slipped at the end of his high school career and really gained little knowledge from these classes. He saw mathematics as numbers and formulas. There was no real use to it.

This view has changed since he has been in the 102 course this summer. he sees alot of usefulness to mathematics. He saw many examples this summer. He said he sees himself using mathematics in the future. He feels like he has gained better interpretation skills from this course.

He really enjoyed the open discussions in class and the fact that he didn't have to take a final exam. He said that he didn't take alot of notes. He like to sit and listen to understand. He said that the two tests he took were easy. He like that. They tested basic skills.

He did have problems with the group situations. The projects seemed vague. They didn't specify what they wanted. His attitude toward the group was that he didn't like to depend on others for his grade. His role was the reality checker. He said that at the first meeting no one would say anything. He ended up taking over as organizer. The other two members gave no input. They had not ideas. There was no chemistry in the group. Finally they ended up splitting the project into parts and everyone working on their own. They especially had difficulty with project 1. They couldn't come up with formulas for the models. They had no idea where they came from. He ended up making up data that sort of looked like the behavior of the different model types. *****MBS NOTE***** I remember this project. The numbers did not fit the models. I ran regressions and programs on their data. I'm not sure if he understood the basics behind the model structure.

As for the calculator, he liked it. He thought it would be alot harder than it was at first. He really liked being able to graph the functions. I asked if he read the written materials. He said that read some, but did not buy the third book. The professor did not talk about it, so why get it. He said he got confused about some of the mathematics in the first book. He said it did not explain itself exactly. He had some questions.

At first he said he could see the whole course being project driven. I then approached him on the subject of student responsibility for learning. He then changed his mind and said that tests were motivational. Without them, students did not care whether they studied, read materials, came to class or not.

Student Interview: August 3, 1993

Sex: Female
Race: White
Age: 18
Year: Sophomore
Major: Political Science (May change to English w/theater minor)
Number of times in MTHSC 102: 1
Other Math Courses at Clemson: none
College courses at Greenville Tech.: College Algebra

This student went to Pope High School in Marietta, GA. She took Algebra I & II, as well as Geometry. Her high school mathematics experience was rather novel. She participated in math class presentations and even bought a graphing calculator (TI-81) in hopes of taking calculus there. However, she graduated a year early and went on to college. Although she was involved in this progressive atmosphere, she had difficulties in mathematics. She said, "I don't understand anything." Yet she really liked geometry and thinks that mathematics is very important to one's education. When asked to define mathematics she said "comparisons of quantitative and qualitative data." and math class was basically a place where you sat and copied down what the professor wrote on the board. HMMM!

I was fortunate to catch this student on the last day of class. She said that her attitudes toward mathematics had not changed during the MTHSC 102 course. She felt that very few people got alot out of the class. She said that most of the students she talked to were completely lost until the day before the test when they would get their review sheets. The tests were your basic "can you use the calculator" type tests. Application came in the form of the projects.

This student really liked the course aim of getting students prepared for the real world. She wishes that more of her classes were application oriented. There was a lot more discussion in this class. Also she really liked having the graphing calculators. They were helpful in interpretation of some of the problems they did. Another positive aspect of the class was the reading materials. She read through all of them and found them very helpful. In fact, she said they were the "best math books I've ever seen." However, she said that there were some people who did not buy any of them.

She felt like she did get something out of the class, but there were several things she would like to see changed. First of all, the "pass/fail" assignments terrified everyone. Also, there were questions asked in the class that they didn't know the answers to. (This really bothered her. I could tell that she was not used to having to think for herself. She was used to regurgitating what she had been told.)

Another problem with the class was the group work idea. She felt that the groups needed to be more homogeneous. The first project was dominated by one person. She said didn't understand that project until after she read the press release he had written. She felt OK with the second project. But, here, the day before the 3rd project's presentation she was having her doubts. She was really apprehensive about what she knew about it. On the whole she said she understood the majority of what was going on but spent most of her time trying to get the third person of the group to understand the basics. She really did not like the fact that the scribe had to do most of the work. The fact that the positions within the group were assigned did not make things any better. She felt that the group members should change positions after each project or that the scribe's job could be split apart some how. Of course the group situation would have been greatly helped if the projects had been clearly worded and were related to something she knew about. (She said that most people did not have general knowledge about interest rates etc.)

Overall, she thinks the class is a good idea, but she feels it needs alot of work. She said she would have felt better if someone would have come in and said, "This is what calculus is. These are the goal for the course." She said that she lost interest in the class and brought another book to read during class.

APPENDIX III-C

MthSc 102 Project 3

Fund Raising Campaign

MthSc 102 Project 3
Fund Raising Campaign

Setting: The math department will be selling T-shirts before the 1994 Clemson-USC football game in order to generate enough money to purchase TI-82 calculators for our future 102 classes. You are such conscientious students and so concerned about providing up-to-date technology for Clemson's future students that you have volunteered to head up this fund raiser. Since several other student groups have also volunteered to head up this project, your team will be presenting your proposal for the fund drive as well as your predictions as to its outcome to a panel of math professors.

Task:

I. Getting Started:

You will need to come up with a slogan and design for the T-shirt. Keep in mind that good taste is a concern. (In fact, your grade will be docked for designs in poor taste.)

Poll at least 100 students (at random) to see what type of demand (as a function of price) may be expected. Use the results from your poll to estimate the total number of T-shirts you could expect to sell to the entire student body. Model a demand curve as a function of price. Keep in mind that your model must make sense for all possible values in the domain.

Consult a T-shirt company price list to determine the costs based on the number of T-shirts you expect to sell.

II. Proceeding:

Use the information gathered in part I to model revenue, total cost, and profit as functions of price.

Determine the point of maximal revenue. Is this the same as the point at which profit is maximized? Why or why not? Which should you consider in order to get the best picture of the effectiveness of the drive? Re-evaluate the number of shirts you may wish to sell. Will this affect the cost you determined above? If so change your revenue, total cost, and profit functions to reflect this adjustment and re-analyze optimals. (If you use your calculator to pinpoint maximums, you must also show how you would find the maximums using first and second derivatives).

Discuss the sensitivity of the demand function to changes in price (check rates of change for $14, $10, $6, and $4). What roll might this sensitivity have played in the number of times you had to re-evaluate the cost before deciding on the correct optimal solution?

Find the point (if one exists) at which profit begins to slow? Would it be wise to continue the drive past this point?

Based on your findings predict the optimal selling price, the number of T-shirts you would have printed, the costs involved, and the number of T-shirts you expect to sell before realizing a profit.

III. Reporting:

A. Write a report for the math department to review concerning your proposed campaign. They will be interested in the business interpretation as well as an accurate description of the mathematics involved. Make sure that you include graphical and tabular as well as mathematical representations of your demand, revenue, cost, and profit functions. (To clarify, have graphs for any functions and derivatives you may use.) Include your calculations as an appendix.

B. Present your proposal and findings to a panel of math professors. Your presentation itself should be restricted to the business interpretation and should employ the use of visual aids. You should be prepared to answer questions concerning the mathematical aspects of your report.

Deadlines:

Information in part I: Th. Nov. 11 in class
Models: T. Nov. 16 in class
Rough Draft: F. Nov. 19 by 4pm
Final Draft: M. Nov. 29 by 4pm
Presentation: T, Th. Nov. 30 or Dec. 2

APPENDIX III-D

MthSc 102 Summaries of Final Project Presentations

Fall 1993

Summaries of Final Project Presentation

Fall 1993

Group 1: November 30, 1993

This group consisted of three males. These students chose a logistics model to represent their student demand data for T-shirts. They concluded that they would sell approximately 300 T-shirts. If they sold to people at the stadium on game day, they could sell more than that. They were asked how they found their maximum value for the Revenue function. They said that they used the solve function on their calculators. What if they didn't have a calculator? The students said that they would graph the derivative function. One student began talking about the characteristics of two functions: s_1– where it "maxs-out" and s_2-where the derivative curve crosses the x-axis.

The group was also asked to talk about sensitivity analysis. One student said that looking at their demand curve around the chosen sell price of $9.25, the curve didn't look too sensitive. The students went on to identify a secant and talked about average rates of change. They then said that sensitivity was concerned where the derivative was steepest.

This group received rather low scores from the graders.

Group 2: November 30, 1993

This group also contained three males. They conducted their poll at the library, dorms, and the student union. This group hoped to sell around 7000 t-shirts, but were only going to buy 1000 at a time. The model they used to make these decisions was a cubic model.

When asked how they found their maximum revenue value, they said they set the derivative to zero. The grader asked how they got their equations. They did so by plotting their data and finding a model curve that best fit this data. They tried cubic, exponential and logistic models. The basis for their decision was the correlation coefficient r^2. They volunteered that the couldn't extrapolate with this model because the cubic function" does funny things." They were asked if it were important to extrapolate and they said it wasn't. Their reason was because if they charged anything beyond $18 (which was their top dollar figure from the poll data) no one would buy the shirts. Also, anything below $2 (their lowest dollar figure from the poll data) would not make them a profit. Finally they were asked about sensitivity. They said that the function was most sensitive at the inflection point.

This group earned rather high marks from the graders.

Group 3: November 30, 1993

This group earned perfect marks in all categories from the graders. Their presentation was very short, yet covered all pertinent information ranging from data collection to model construction. They decided that they would maximize their profit if they sold their shirts for $9.97. This group even designed their own shirt and had one printed up for their teacher.

An interesting question given to this group was if the revenue function and the profit function achieved maximums at the same value. The students said that they wouldn't. When asked why, the reply was that the cost function caused the difference. They were also asked about sensitivity. The students said that there was very little change in the graph between the values of $9 and $10. They said the flatter the curve, the less sensitive the demand is.

Group 4: November 30, 1993

This group was graded by different instructors than those above. The graders were perplexed in grading this group because of their model choice. After plotting their data, which they obtained by polling 100 students in front of the library, they decided that they would use a linear demand model. The sell price would be $10.33. They said that 58% of the people polled said that would by a shirt for at most $10.00. Their target population would be 10,000 students (there are approximately 13,000 students on campus) and they hoped to sell around 5000 shirts. These students were not able to make a connection between their profit function and its derivative. However they were able to say that sensitivity was the same everywhere with the linear model. This group was finally asked why they thought their data yielded a linear model. They were unable to give much of an answer for this.

Group 1: December 2, 1993

This group said that their target population was 10,000 people. From their poll, 81% would by a t-shirt for at most $8. From this they implied that if they sold at the game, they could sell around 8100 t-shirts. However, they projected that they would sell around 6000 shirts. This appearance of this number was not explained.

When asked about sensitivity, they replied that it was a "rate of change." When asked why it was useful, the students replied that they wanted to know how fast things were going to change and where those changes would occur. The number, "6000" mentioned above, was brought up. The students said that their logistic model gave them a limiting value of 6050. Finally, they said that they could maximize their revenue if they sold shirts for $8.55. However, they couldn't make the connection between the revenue function and the profit function.

Group 2: December 2, 1993

This group seemed very confused. From their poll, they said that selling their shirts for $10 would maximize profit. They put up an overhead that said they had a limit value of 6050 for revenue; however, the next transparency showed the graph of a quadratic model. This was puzzling. They went on to show their model equation which was a logistic function.

I'm still not sure how they got their quadratic graph. But that is not the most important point. The most astonishing fact is that they did not realize that there was a problem. It made me wonder. Since most of the time was spent trying to find out why they had a discrepancy, there was little time left for any other questions. However, the students did get a chance to talk about sensitivity of their data. Again, there seemed to be a great deal of confusion. They said that sensitivity increased as price increased. However, they said that the function was most sensitive at $10.50.

It is hard to believe that all three people could be so lost.

APPENDIX III-E

MthSc 102 Project
Guidelines for Graders

MthSc 102 Project 3 - Guidelines for Graders

Thank you for agreeing to help us out with grading the MthSc 102 project presentations.

You are scheduled for Tues 11/30 **from** 9:30 / 2:00 **to** 10:45 / 3:15
in room O-106.

You will be grading along with Laurel Carpenter at 9:30
not sure at 2:00

One of the two graders in each room will be given a grading sheet listing the members of the groups presenting. You will need to call each group in when it is their turn (They should be waiting in the hall).

You will also be given the written report turned in by the students. Written reports will be graded by the teacher, and are provided solely for your information. Feel free to question the students about anything in the report as well as in the presentation. **The students have been told that questions will be asked of members of the group other than the presenter.**

Please allow the group to give their entire presentation before asking questions.

On the grading sheet

A. in the first column to the right of the student's name, place a checkmark if the student is present.

B. each group gets assigned one grade. Don't grade them individually.

C. in column E assign points out of 15 for the completeness and accuracy of the content of the presentation. Minimum content requirements are:

1. A discussion of demand, revenue, cost, and profit models.
2. A discussion of the prices that maximize revenue and profit and how those prices were obtained.
3. A discussion of the sensativity of demand to changes in price.
4. An outline of the details of the t-shirt campaign :
    - design of shirt
    - selling price
    - # of shirts they expect to sell
    - where/how they plan on marketing the shirts
    - profit they expect to make

D. in column F assign points out of 5 for the professionalism of the presentation.

E. in column G assign points out of 5 for how well group members other than the presenter answered questions to your satisfaction.

F. column H is for the total score and column I for any comments you wish to make about the presentation for the benefit of the teacher. (the presentation is 25 % of the project grade.)

Please return the written reports and grading sheet to Cindy Harris.

BEST COPY AVAILABLE

APPENDIX III-F

MthSc 207
Portfolio Instruction

PORTFOLIO

Your portfolio **should be organized** to show me your progress (or lack of progress) through MthSc 207. It should be in some type of folder with your name clearly indicated. As an incentive to pay careful attention to the organization, *your portfolio will be worth 15 points of your score on the final examination.* The portfolio must be turned in no later than 4 p.m. on Friday, December 10. Your portfolio should include, but is not limited to:

1) All of your original journal pages (I do not want notebooks - tear the pages out of the notebook if you have a notebook for your journal.)

Attached to each of the following should be a sheet of paper containing a complete explanation of why you chose the test, quiz or project that best fits the description. Either the original test or quiz or a xeroxed copy should be placed in the portfolio along with the explanation. Do not let your grade entirely influence your decision.

2) The test, quiz, or project that you feel presented you with the biggest challenge of your your ability.
3) The test, quiz, or project that you feel best showed you a use of calculus in the "real" world.
4) The test, quiz, or project that you wish you had never seen and would like to forget.
5) The test, quiz, or project that you would have been lost on without the calculator.
6) The test, quiz, or project on which the calculator did not seem to help much at all.

Your folder should also contain an estimate of your numerical score (out of 100 points) on the final examination. You should take your four (numerical) test scores as the y-values and the numbers 1, 2, 3, and 4 as the x-values of four data points.

7) Find the best fitting model *that makes sense* for a prediction of your score on test 5, the final examination. In addition to the available models in your calculator, you can use the average of the four test scores. You should include in a discussion of why you chose the score you did
   - the numerical estimate of your score on the final exam clearly indicated
   - a scatter diagram with the model you choose drawn on the scatter diagram
   - the reason(s) why you chose the model you did and rejected other available models

P. S. If you correctly, within 2 points, predict your exact numerical score on the final, I will add 5 points extra credit to your final exam grade.

MthSc 207
GRADE SUMMARY

Name albert

Total Points on **Quizzes*** out of 70 points
(Lowest quiz score has been dropped - 10 points on quiz 9 still to be added in to Q total)

$Q =$ 86

Total Points on **Journal** out of 75 points
(25 points on last journal entries still to be added in to J total)

$J =$ 65

Total Points on four **Major Tests** out of 400 points

$T =$ 354

Total Points on **Projects** out of 60 points
(40 points on project 3 still to be added in to P total)

$P =$ 60

Best estimate of your grade going into the final exam is A

I will compute a numerical average using the formula

$$\underset{8.6}{\frac{Q^*}{80}(8)} + \underset{17}{\frac{P^*}{100}(17)} + \underset{43.9}{\frac{T+J^*}{500}(50)} + \underset{:25}{\frac{F}{100}(25)}$$

to help me determine your course grade. Letter grades listed on the MthSc 207 course description will correspond to the numerical averages 90-100 A, 80-89 B, 70-79 C, 60-69 D, and 0-59 F.

Note: The *'s indicate that points on remaining work will be added to the totals Q, P, and J given above. The letter F in the formula is your final exam grade. Good luck!

Your calculator and manual must be returned no later than the time you turn in your final exam unless other arrangements are made with me. If I am not in my office, you may return your calculator to the secretary in O-101 Martin Hall. Be certain that your name is on a slip of paper inside the calculator and manual if returned to the secretary.

* *If you turn in your calculator manual to me in my office before you take the final exam, I will add 5 points extra credit to your quiz total. If I am not in my office, slip the manual under my door with your name on a slip of paper inside the front cover of the manual.*

According to University regulations, I cannot post grades or give out information on grades out over the phone. If you wish, I will mail your exam grade and course grade if you slip a self-addressed stamped envelope or post card under my office door.

L. B. Fetta

The MthSc 207, Section 3, final examination will be in Room E 305 Friday, December 10, from 1 - 4 p.m.

Office Hours: Wednesday, Dec 8, 10-12 am and Friday, Dec 10, 10-12 am

APPENDIX III-G

Sample Portfolio Entries:
Business Calculus II Students
Fall 1993

Five Samples Given:

- Sample of student journal for the semester (Item 1 on Portfolio requirement sheet)
- Sample from Item 2 on Portfolio requirement sheet
- Sample from Item 3 on Portfolio requirement sheet
- Sample from Item 4 on Portfolio requirement sheet
- Sample from Item 7 on Portfolio requirement sheet

Journal

- Matt Watters

8-25-93

Today we put our groups answers on the board. They were consistent with all of the other groups. We talked about those problems about the slope of the tangent line for the rest of the class. We got a worksheet to work on for homework

8-26-93

I read chapters 1 and 2 in the book. Most of the functions and lines were review. The derivitives were a review also but it turned out that I had forgotten everything that I had learned about derivitives. Then I worked the first 2 problems off of the worksheet. It was a good review from last semester. I entered the data into the data cards and then used the scatter diagram to graph it. I set the range and then used regression to choose the best formula. Then I looked over the derivitive section some more and went to bed.

8-27-93

In class we took the first problem from the worksheet and worked on it for the whole class period. We learned to find the best fitting line, using the scatter diagram and regression. We also learned how to →

represent our data. The class moved along very slowly. I don't remember going this slow last semester. I guess it was because this was review to some of us but ~~[illegible]~~ new to alot of people. I ended up getting bored so I worked some more problems on the worksheet and then played toothpick.

Right!

8-30-93

We basically did the same thing as we did on Friday. We covered 2 more examples on the worksheet. Class is beginning to move a little faster but I'm still getting bored with it. We learned to use Polyfit & Graphit also. This was also review but some of it was unclear. I had to relearn that m=2 is exponential and m=3 is cubic. We had to do the worksheet and tell whether the data was a parabola or cubic.

I hope i
will spe
up soon

8-31-93.

I finished the worksheet for the next class. Everything began to come back to me. I took a while to do but it was a good review. I couldn't remember the formula for a parabola.

9-1-93

In class, we went over the worksheet and used the quadratic formula. Most of my answers were wrong. I figured out what I was doing wrong. I was forgetting to put the data in the D-matrix. My other mistakes were just stupid errors. I feel pretty good with where I am. Sometimes I still get lost in class. It's usually my fault though for daydreaming.

9-2-93

I just reworked 2 problems on the worksheet that I missed. Everything came out all-right this time.

9-3-93

Today in class, we were given a review sheet for derivitives and the average rates of change. I feel like I'm pretty weak in the area of solving derivitive algebraically. Most of these formulas were foreign to me. We were told not to worry about the chain rule. Our homework assignment is on the bottom of the page.

9-5-93

I just got finished with the homework for Monday. I couldnt figure out # 1.b on the worksheet.

$$\frac{dG}{dv} = {}^{-}0.25 \text{ When } v = 55$$

9-6-93

We used more graphs to represent the data. We also used the derivitives to show average rate of change and instartaneous rate of change. - No problem. The take home quiz is due on Wednesday.

9-7-93

Just finished the quiz. I wasn't sure about alot of my final answers. I did O.K. in using the calculator. I had the most trouble in interpreting the data...

9-8-93

We worked with more derivitives on the calculater. We found some assumed points on our data. We used the graph mode (y^1) and (y^2) to graph the derivitive by pressing Math-Calc dx/dy (y'). This graphed the graphs for us. We found the max, min, & x & y intercepts. Then we turned in the quiz.

9-10-93

Our quizzes were returned. I made a 14 out of 20. There was alot of red on it. Some of my errors were pretty stupid. This really scares me though. I thought I knew what I was doing. I worked with the girl beside me with a problem off of the board. We told where y is increasing, y is dec, x-value of max, x value of min., y is concave up., y concave down, x-intercepts. I missed most of these but I think I know why

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9-11-93

I looked over the quiz. I know we're supposed to put corrections in here but mine were already on the page. Most of my mistakes were on explinations. I also did the home work of the graph and its derivitive. I couldn't figure out how to sketch its derivitive.

9-13-93.

We reviewed the homework assignment. I missed the second part where we had to graph the derivitive and the sketch on the back side. We then took a quiz much like the one that was a take home. Then we were given a test review sheet.

9-14-93 Test Review

Today, I spent about $4\frac{1}{2}$ hours working problems for the test. I found so much that I was unsure about. How does the y-values tell whether it will be a line, exp. . . . . I forgot all of those. Noone I called knew either. I worked my way through the review test well. It just took a lot longer than I will have on the test. That's the biggest worry that I have. Well, its time to take the test.

Matt Waters

9-15-93

Today we took the first Test. I'm not sure how I did. I answered all of the questions but I just didn't feel good about any on the back side of the page.

9-17-93

We got the tests back today. I made a big fat F. I can't believe some of the problems that I missed. My mind must have just gone blank or something. What in the world am I going to do, I am already digging a huge hole for myself. My quiz average is pretty bad too. After we went over the test, we didn't much time left in class. We're supposed to read over the Chain rule on our derivative worksheet for the next class. Things look like there going to get alot harder. I need this weekend badly.

9-20-93

Today we worked the homework problems from the derivitive worksheet and from the book on the board. We are using the Chain rule. This is really foreign to me. I don't really have a clue whats going on. I need to get help from my friend in the class and work some problems on my own. This stuff is really frustrating me. I don't know if its just me who is lost a are there other people who don't know what the heck is going on.

9-22-93

Today we did some more problems using the power rule. I am pretty lost with a lot of this stuff. I didn't have time to work any problems last night for homework so that is probobly why. I don't understand the terminology that goes along with all of this. I hope to figure it out over the weekend.

9-24-93
Friday

Today we finished up with the sum rule. I guess I understand what is going on but it is still kind of shaky. I still don't know what $\frac{dx}{dy}$ means and what $\int$ stands for. I know its the integral sign. Is that the same as differentiation? No =

Everyone that I usually talk to is lost so they can't really help me out too much. I need to come by your office but our schedules conflict. (Make an appointment.)

9-27-93

We were given the stop watch program to do today. It is due sometime next week. We also started with the log rule. This just confuses me more. I am so lost that I can't pay attention any more. I am afraid to ask questions because I get lost on the second step of the problem. I guess its my fault for not asking though.

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10-4-93

We are working with the other rule. Its pretty easy. $\int e^u du = e^u + C$ I don't know what the name for it is but this is the easiest one of them all for me. I can understand some of the easier homework problems, but the harder ones, I still can't do.

10-5-93

Tonite I hurt my knee in a soccer game. I spent the entire night in the hospital. I doubt I will make it for class tomorrow.

I didn't know you were on the team. Or are you?

Sorry!

10-6-93

Can't walk or get around. I'm on crutches & a knee brace. Hurts to move. My moms coming from home to get me tomorrow and take me home to see a specialist,

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10-5-93

Missed class again for knee, I'm at home. I left a message with the Math dept. secretary for you. I couldn't find you though.

10-11-93.

Today is my first day back to class in a week. I am pretty lost. I didn't get any studying done at home. I was pretty drugged up most of the time. We took a quiz. I'm sure I failed it. I will get help on whatever I missed. Good!

10-13-93

We got the quiz back today. I made a 1 out of 10. Here are the problems I missed

2. $\int \frac{2\,dx}{5\sqrt{x^3}} = \frac{2}{5}\int x^{-\frac{1}{3}}\,dx = \frac{2}{5}\frac{x^{\frac{2}{3}}}{\frac{2}{3}} = \boxed{\frac{3}{5}x^{\frac{2}{3}} + C}$

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3. $v = 320 - 32t$?

$t = 10 = 1600\text{ ft}$

4. $\int (x^2 - 2x)^4 (x-1)\, dx = \frac{1}{2} \int u^4\, du$

$u = x^2 - 2x$

$du = 2x - 2\, dx$

$= \frac{1}{2} \int \frac{x^2 - 2x^5}{5} + C$

$\frac{1}{10}(x^2 - 2x)^5 + C$

I know how to work the problems O.K. but the word problems give me alot of trouble.

10-15-93

I missed class again today. I had an MRI done at Oconee Hospital. This was the only time that they could see me.

10-20-93

Today we got the quiz back that I missed on Friday. I could have done most of those problems I forgot that we had a quiz on Friday. We worked through alot of problems from the test review sheets. I'm still ok on most of the stuff. I make alot of mathematical errors though. We were given a take-home quiz to do.

10-22-93

Today is the last classday before the test. I've worked through about one half of the problems on the review sheet. The answers from the library help out alot. This is going to be a great homecoming weekend studying calculus. The problems toward the end of the review sheet are pretty difficult. The Chain Rule gives me alot of problems also. Only in the substitution part

10-23 & 10-24/93

This weekend I spent the entire time working all the problems on the review sheets. I think I have a good understanding of what's going on with each problem. I still make a lot of stupid errors though. I hope partial credit will be given if I just make an error in my math. Overall, I feel ready for the test and I hope I can better my performance on the last test. If I can't, I'm done but I've tried my best.

That's a good start! + +

Monday, October 25.

Today we took the 2nd test of the semester. I felt like I knew the problems on the front of the page well. The word problems on the second page may have been my downfall though. I didn't have any time left to check over the test

Wednesday October 27,

We didn't have our tests returned today. This is the last day to drop the class. I am going to get my test back before I decide. We began working with exponential growth and decay using the formula $y = Ce^{kt}$. We defined each of these variables and then used the solver to solve the problems by pluggers the formula in. We learned how to use the solver and how to always get the formula where we are solving for only one variable. .... I went and got my test back after class today. I am very disappointed that I made a D. I never do as well as I think. I an not going to drop the class though. I am going to stick it through. Our skin projects are due on Wednesday.

Good for you!

Thursday Oct. 28

Our projects are due tommorrow and up to now I have done all of the work. Our group seemed to never be able to meet nor did they ever care. Now most of them dropped the course. No one will ever return my phone calls. I did the project myself and put everyones names on it. It really made me very mad.

Thanks for letting me know—

Friday Oct. 29.

We worked more ~~projects~~ problems with exponential growth and decay. We went over the homework problems very thoroughly and then had a group quiz at the end of class. I turned the project in at the end of class. No one (in my group) was interested in whether we had one or not. It probobly wasn't as good as the other groups but it was the best that I could do by myself.

I'll try to put you in a different group next time

Monday, Nov. 1

We worked some more problems with the exponential growth and decay. This is the first time that we talked about half life though. I'm not too clear what half life is in the big picture. I feel pretty comfortable with most of this. We will be doing a large number of problems again this week for 2 quiz grades. The problems will be given to us on Wednesday. There will be no class on Friday.

Wednesday Nov. 3

We worked some more exponential growth and decay problems. I can tell this is very important since we have spent so much time on it. We will be having a test either next Wednesday or Friday. I understand this pretty well. Getting the formulas narrowed down to solve for one variable is where I get messed up. We had a quiz today. I got messed up on it, I know that stuff but it went blank. I want to get some names of some tutor's from Mrs. Fetta today. I think that will really help me alot Did you find one

November 5

Class was cancelled for today but we were given an assignment to do for today that will be due on Monday. It will be worth 2 quiz grades so I have to do good. I had a tough time figuring out how to do the problems. I eventually figured them out though. First you have to integrate. Then you just have to go back + plug the two values in and subtract the latter from the first. I was able to do the strait problems but I didn't know where to begin on the word problems. Tell the tutor that -

November 8.

We turned in the Homework quiz that was due today. Then we were given six formulas that are used to calculate interest earned on different accounts. We worked through several examples of these and plugged all of the formulas into the solver memory. I understand all of them. The trouble that I have is deciding which one to use. We also did some where we had to take the initial amount, add the interest, make a payment and do it all over again.

November 10

Today we began class by running the slosh program. My calculator drew a flat line every time. Then we got our quizzes back from Monday. I made a 15 out of 20. 75% I also made a 1 out of 10 on the daily quiz that we took on Monday. I just want to throw my books out of the window now. Then we worked with the formula $\left(\frac{1}{b-a}\right)\left(\int_a^b \overset{f(x)}{\cancel{\text{equation}}}\, dx\right)$

Hangin there!

The test is on Friday.

November 11

Today I ~~went~~ and picked up the answers to the ~~od~~ even numbered problems on the test. Then I went to see a tutor that I got from the Math office. This was the worst tutor I have ever seen. She couldn't do one single problem off of the review sheet. I left there & called my friend Eric and he answered all of my questions. I'm ready. Give me the test.

→ Come See m with + list of nam

Great!

Nov. 12 TEST

Matt, I need the quiz corrections.

November 15,

Today we got back to class and had our third test returned to us. I finally made a good grade on a test. We did a little bit with simpsons rule today, to tell the truth. all, I did in class was let my head swell up thinking about the last test.

Wednesday Nov. 17

We just kept on with Simpsons and some left and right sum rules. I am pretty lost as far as working through the problems on my own. Its just that I see it when the teacher does them on the board, it makes sense but when I do them alone, I get lost. I guess I need some more practice.

Friday Nov. 18

I missed class today because I I had to get blood work done at Oconee Hospital for surgery on Monday.

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Monday, Nov. 22

Surgery today. I won't be back to school until Wednesday.

Wednesday, Nov. 24

I was discharged from the hospital today. Looks like I'm going to have a great T-giving break. I have alot left to make-up for the test.

Monday, Nov 30

Thank goodness Mrs. Feeta let me pospone test until tomorrow. I cant sit up without getting migranes. Its from my Epidural from surgery.

Tuesday Nov 31,

I took the test today. I feel good about it. I'm pretty sure I got a B. Eric tutored me on Sunday night. Can't wait to get it back

Wednesday Dec. 1,

Last day of class. I watched everyone turn in there paper copter projects. We then took a quiz that was pretty hard. Then we got our grade summeries, I have a C. I have worked hard just to recover from the first test. I wish I wouldn't have dug such a deep hole for myself. I feel O.K. about the Exam, Mrs. Tella has helped us out alot with a few extra credit points.

Dec. 9

Studying hard for my final. and also putting the final touches on my Portfolio. I can't wait to get this last Math Class out of the Way. If I happen to blow it tommorrow, Please Help Me, these has been the hardest semester w/ my knee.

thanks

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14/15

2. The test, quiz, or project that I felt presented me with the biggest callenge of your ability.

The project that I felt presented me with the biggest challenge of my ability was the journal. I know that this sounds crazy, but I chose this because for one thing it was a big challenge to make myself remember to write in it every day. The question asked not only what was the biggest challenge, but also the biggest challenge of my ability. Writing in the journal was not hard, but I felt since we had to put all corrections to the tests and quizes in it, I thought it was a challenge. After all, if I didn't know how to do it on the test than it has to be a challenge to figure out the correct answers. At times I thought this was very hard because I found myself looking through my book and notes trying to figure out how to correct the problems. I'm glad we had to do this because it made me learn and better understand the problems I had missed.

3. The first group project that we did this year that involved calculating the total area of skin on the body showed me the best use of calculus in the "real" world. Finding the way that I was going to go about finding the formula showed me how it was so important. I called a friend of mines father in Concord, NC to ask him how he would go about finding something like this. He knew and remembered how from his years in med school long ago and his many years of practicing. He didn't tell me how to do it, he told me where to find it and the many uses that he has for the idea which we studied. Dosages on prescribed medicine are determined from a formula similar to this. Seeing that he uses every day something that I would probobly think was just another assignment really showed me its importance in the real world.

How Much Skin Do You Have?

Octover 29, 1993

Math 207

Group #6

Matt Walters

Jeffrey Harmon

Donnie Skelton

Russell Compton

HOW MUCH SKIN DO YOU HAVE?

To measure how much skin covers the body, I used geometric shapes to estimate the surface area of the body. This is how we estimated the surface area and amount of skin on the human body. We broke the body down into geometric shapes. The head is a sphere. Then we have five main cylinders which are the arms, legs, trunk, fingers and toes. We need to find the surface area in each of these and add them together to get the surface are of the entire body. To measure the surface area of each sphere, we use the formula ( ). To measure the surface area of the head, which is a cylinder, we used the formula ( ).

body part	length	radius	surface area
right arm	*66.7cm	4.4	1843
left arm	66.7	4.4	1843
right leg	80	7.62	3828
left leg	80	7.62	3828
trunk	69.9	20.32	8919.9
neck	8.9	9.43	527.1
fingers (average)	16.4	.95	971.0
toes (average)	12.4	.49	382.0
right foot	19.05	5.08	607.7
left foot	19.05	5.08	607.7

*all data are measured in centameters squared

These measurments are for the cylinder shaped body parts. The total surface area for these is 23357.4cm. Then we take the surface area for the head by taking the radius (9.5) and squaring it and then multiplying it by 4 . Then we add this to the previous total which gives us the overall total surface area for my body. 24490.9cm .

A person's weight more closely corelates to the amount of skin covering that person's body(Dr. Fortney, interview). The heigth has little to do with the surface area other than helping us find the volume of the area which we need to find the surface area. weight corresponds with mass which corresponds with the surface area.

Fourier's law states that the rate of loss of heat it proportional to the surface area. but the surface area is roughly proportional to m^2/3 (where m is mass. commonly called body-weight). whereas the heat capacity of a body is proportional to the mass itself. Therefore. the rate of cooling of a body will be proportional to m^2/3 /m=m^-1/3: since m is proportional to l^3. where l is the linear dimension. as the size of the body decreases. its rate of loss of heat will increase. since it is inversely proportional to the size as prepresented by l (Richards. 25)

The elderly tend to have a higher percentage of skin when compared to the mass of their bodies. This is caused by the deterioration of bones and muscles while the skin does not leave. It just forms wrinkles.

Works Cited

1. Richards, S. A., *Temperature Regulation* (London: Wykeham Publishers, 1973), pg. 25

2. Interview with Dr. Sidney R. Fortney, Internal specialist, Concord, NC, 21 October 1993.

#4

I would wish that I had never seen and would like to forget this project. the reason why I didn't like the skin project is because I could never figure out a pupose for it. All it really did was get me very confused. First of all I thought this project was a little above our heads. It didn't include anything we were doing in class, and I couldn't think of one good reason any of the information we were useing and found go do any good. I found no purpose for it in the "real" world, and our answer had to be so rough, that it couldn't have even been close. We did however all work together and documented all our findings. Maybe that was the purpose of this project?

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MthSc 207 Project 2

This project will be worth 50% of the project grade. It is due in class no later than October 29th, 1993. More details on the project, including suggestions for progression and preparing the written report, will be distributed next week.

This project is a group project, and all members of each group will receive the same grade. It is assumed that each group member will participate equally in the design of the project, data collection, and writing the final report.

You should research whatever you do not know in the library. All references used should be given in a bibliography at the end of your final report. You can use the advice of persons outside this class as long as they are credited in your references.

HOW MUCH SKIN DO YOU HAVE?

Preliminary Thoughts

The amount of skin you have has a direct bearing on the rate of heat loss from your body. A major part of this project is to determine a way to measure the amount of skin covering a person's body. Once this has been decided, you will collect data in an effort to decide:

- Is a person's height or weight more closely related to the amount of skin covering that person's body?
- Can you formulate an equation relating the amount of heat loss to the amount of skin covering the person's body?
- The answers to any other related questions you feel are pertinent.

Your group is given below. Your group should meet before Wednesday, October 6 to discuss this project and formulate thoughts on the following question that will be discussed in class on October 6.

How can you measure the amount of skin on a person's body? Which age people should you use to obtain the measurements? Should you use both men and women, or does the amount of skin covering a person's body differ significantly due to the sex of the person?

Group # 3

Justin Ross	653-4335
Rob Graham	653-9498
Michelle Shirer	858-8484
Kim Brannon	226-7385
Todd Albert	858-3063

DANIEL SCHMIEDT

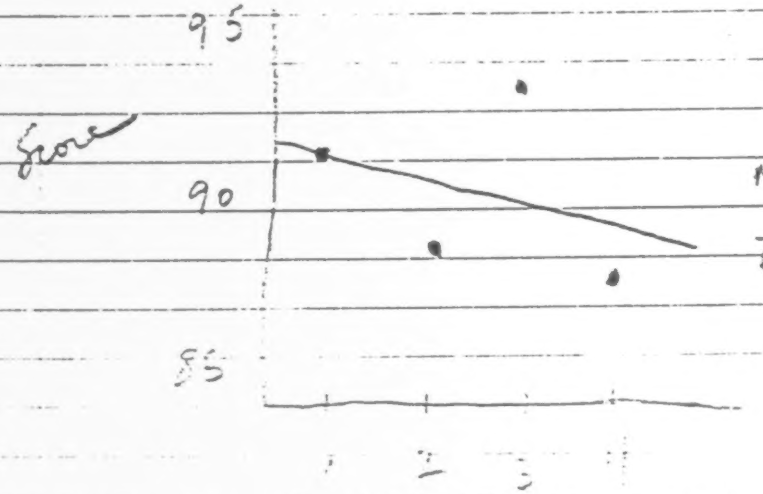
*DISREGARD

MORE REALISTIC, NOT WELL FITTING

PORTFOLIO #7

A CUBIC MODEL FIT WELL, BUT GAVE AN EQUATION THAT GAVE UNREALISTIC PREDICTIONS. IT IS NOT REALISTIC FOR SOMEONE WHO HAS BEEN MAKING A'S AND B'S TO SUDDENLY MAKE A E!.

ALTHOUGH THE R VALUE WAS NOT CLOSE (−.338165061) FOR A LINEAR MODEL, THIS MAKES THE MORE LOGICAL SENSE than the cubic. IT MAINTAINS



Still, there's no reason for my scores to be DECREASING...

$y = 9[illegible] - .7x$ For x = [illegible], $y = \boxed{86}$

DANIEL SCAMIEOT

DISREGARD

[illegible] BY —

PORTFOLIO #7 [illegible]

TEST SCORES -

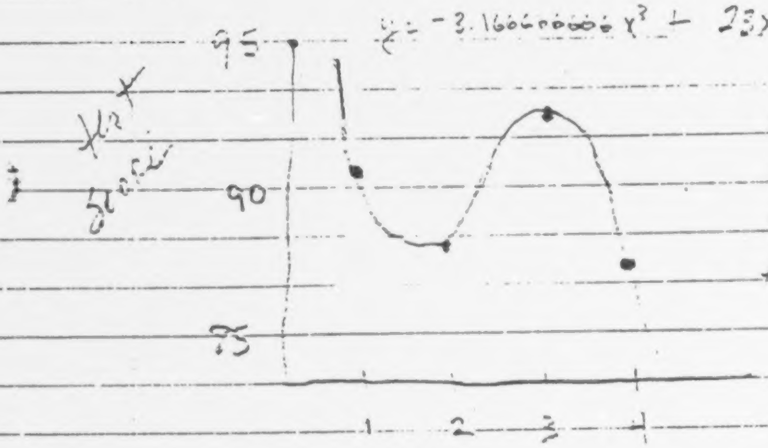
91 > 3 > 8
58 > -5 > -11
93 > 6
57

[illegible] [illegible]

(differences)

NO CORRESPONDING MATCHES, LINEAR, QUADRATIC, INSUFFICIENT DATA FOR CUBIC USING THIS METHOD.

$y = -3.16666666x^3 + 23x^2 - 49.8333333x + 121$



at x = 5, using this model, I should get a $\boxed{51}$ on this test... [illegible]

test #

T1 = 91
T2 = 58
T3 = 93
T4 = 57

CONCLUSIONS

I COULD NOT DETERMINE A TYPE OF MODEL FROM THE DIFFERENCES. BECAUSE THERE WERE SO FEW POINTS, I COULDN'T DETERMINE A 3rd DIFFERENCE TO CHECK FOR CUBIC. SINCE THE DATA WENT UP AND DOWN, I COULDN'T THINK OF ANOTHER MODEL [illegible] THAT WOULD FIT THE DATA. THE MODEL IS ACCURATE [illegible] POINTS BUT [illegible] INACCURATE [illegible] BEYOND THEM. [illegible]

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* USE THIS ONE FOR 5 pts EXTRA –

PORTFOLIO #7

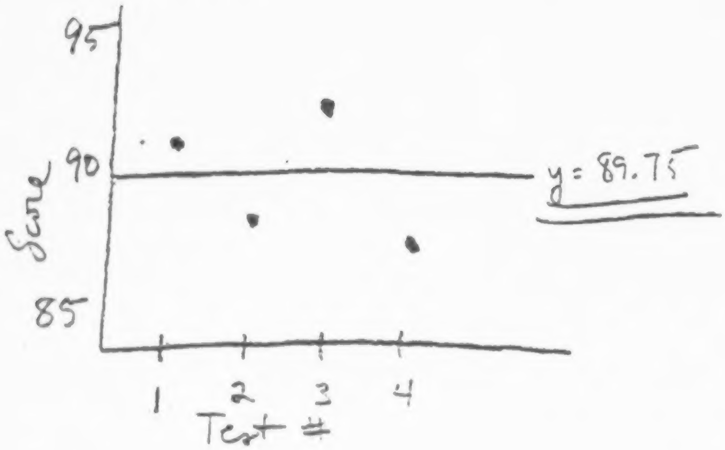
MOST REALISTIC MODEL

$$\text{AVERAGE} = \frac{T_1 + T_2 + T_3 + T_4}{4}$$

$$89.75 = \frac{91 + 88 + 93 + 87}{4}$$

Round up ↓

90 = MOST PROBABLE SCORE



$T_1 = 91$
$T_2 = 88$
$T_3 = 93$
$T_4 = 87$

Although the average does not fit the points well, it represents the most realistic model. My scores didn't vary more than 3 points from this average, so it would be a pretty good bet that my final would fall within a couple of points of this average.

Note: See also other pages to see why I chose this model over some other possibilities.

APPENDIX III-H

MthSc 102 Doubling Time
Project Description

Project 5

DOUBLING TIME

Setting: Dr. C. G. Bilkins, nationally known financial guru, has been criticized for giving false information about doubling time and the rule of 72 in seminars. You have been hired to provide mathematically correct information for Dr. Bilkins to use in future seminar presentations.

Task: Doubling Time is defined as the time it takes for an investment to double. Doubling time can be calculated algebraically from the compound interest formula $A = P(1 + \frac{r}{n})^{nt}$ or the continuously compounded interest formula $A = Pe^{rt}$. An approximation of doubling time can be found by dividing 72 by 100r. This approximating technique is known as the ***Rule of*** 72.

I. Evaluating Doubling Time Rules:

Construct a table of doubling times for interest rates of 2% - 20% (in increments of 2%) when interest is compounded annually, semi-annually, quarterly, monthly and daily. Construct a table of doubling time approximations for interest rates of 2% through 20% when using the rule of 72. Construct similar tables for rules of 71, 70, and 69. Examine the tables and determine the best approximating rule for interest compounded annually, quarterly, monthly, and continuously. Justify your choices.

For each compounding listed above, compare percent errors when using the rule of 72 and when using the rule you chose.

Percent error = 100 * (estimate − true) / true.

Comment on when the rules overestimate and underestimate and which is preferable. The assumption of this report is that the rule of 72 is not the best approximating rule in most cases. If this is true, why is it commonly used?

II. Evaluating Sensitivity of Doubling Time:

Use a technique discussed in class to estimate rates of change of doubling time at 2%, 8%, 14%, and 20% when interest is compounded quarterly. Interpret your answers in a way that would be meaningful for Dr. Bilkins who knows nothing about calculus. In particular Dr. Bilkins is interested in knowing how sensitive doubling time is to changes in interest rates.

III. **Reporting:**

A. Prepare a detailed written report for Dr. Bilkins in which you discuss your research and findings from parts I and II. Be sure to discuss whether Dr. Bilkins should continue to present the Rule of 72 or should present other rules depending on the number of times interest is compounded.

B. Prepare a document for Dr. Bilkins' speech writer to insert into the seminar presentation (which Dr. Bilkins reads from a Teleprompter). It should be a brief summary of how to estimate doubling time using an approximation rule along with a statement about the error involved in using the approximation. Keep in mind that Dr. Bilkins wishes to avoid further allegations of misinforming seminar participants. Also include a brief statement summarizing the sensitivity of doubling time to fluctuations in interest rates. Include the document in your written report.

C. Prepare a brief presentation (5 minutes) of your study. You will be presenting to Dr. Bilkins and the speech writer. Your presentation should be only a summary, but you need to be prepared to answer any technical questions which may arise. One person from the group who has not yet done a presentation should do the presentation. All group members must be present and must be prepared to answer questions.

APPENDIX III-I

MthSc 102 Super Highway
Project Description

Project 4

SUPER-HIGHWAY

Setting: The European Communities have decided to build a new super-highway that will run from Berlin, through Paris and Madrid, and end in Lisbon. This super-highway, like some others in Europe, will have no posted maximum speed so motorists may drive as fast as they wish. There will be three toll stations installed, one at each border. Because these stations will be so far apart and motorists may not anticipate their need to stop, there has been widespread concern about the possibility of high speed collisions at these stations. The concern is so great that there have been numerous editorials written protesting the installation of the toll stations and a private interest group has lobbied to delay the building of the new super-highway. In response to the concern for safety, the Committee on Transportation has determined that there should be some flashing warning lights installed at an appropriate distance before each toll station. Your firm has contracted to study known stopping distances and to develop a model from which to predict where the warning lights should be installed.

Task:

I. **Getting Started:**

Find data showing stopping distances as a function of speed and, cite the source for the data. Present the data in a table and as a graph. Find a model to fit the data. Justify your chose by discussing why other models were not chosen. Consult someone who could be considered an authority to determine whether the model will hold true outside of the data range before using it to extrapolate. Consult a reliable source to determine probable speeds driven on such a highway. Based on your model, make a recommendation as to where the warning lights should be placed. Keep in mind that the Committee on Transportation does not wish to post any speed limit signs. Justify your recommendation. Make suggestions as to what other precautions could be taken to avoid accidents at the toll stations.

II. **Proceeding:**

Find rates of change of your model for at least three speeds, one of which should be the speed you believe to be most probable. Interpret rate of change in this context. Would underestimating the probable speed have a seriously adverse affect? Support your answer.

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III. **Reporting:** (Bear in mind that you are reporting to Europeans who will wish to see all results in metric. However, since you work for an American based company you must also have the English equivalent.)

A. Prepare a written report of your findings for the Committee on Transportation.

B. Prepare a brief (5 min.) presentation of your findings. You will be presenting to members of the Committee on Transportation. One person in your group should do the presentation, but all members must be present and be prepared to answer questions.

C. Prepare a press release for the Committee on Transportation to use when it announces the implementation of your safety precautions. The press release should be succinct and answer the questions Who, What, When, Where, and Why. Include the press release in your report to the Committee.

APPENDIX III-J

MthSc 102 Project
Grading Sheet

Grading Sheet for Projects 4 and 5: Oral Presentation

Instructor: ______________________ Grader: ______________________

Group: a. ______________________ (names)

b. ______________________

c. ______________________

CONTENT (15 points)

(3 pts) Introduction _____

(3 pts) Use of Visuals _____

(3 pts) Understanding _____

(3 pts) Conclusions _____

(3 pts) Communication _____

PARTICIPATION (6 pts)

(2 pts) member a. _____

(2 pts) member b. _____

(2 pts) member c. _____

Total Score (max = 21 pts) ________

GRADE (Check one):

Superior (18-21) ________

Satisfactory (12-17) ________

Unsatisfactory (≤ 11) ________

APPENDIX III-K

MthSc 102
Project Summaries

Final Project Presentation Summaries

Spring 1995

Each grader was given the responsibility of assessing approximately four groups in an hour's time. These groups were not arranged according to topics, so it was possible that the grader would have a mixture of the two projects represented in each of these hours. I have chosen a three samples for each project. The time next to each group number will represent the hour session in which they presented.

Doubling Time Project:

Group 3: 1:25 p.m.

This team of three females presented a table that recorded the actual doubling times for various interest rates. Another table was given that presented all the approximations for Rules of 69, 70, 71 and 72. Their conclusions were as follows: Rule of 70 was the best approximation for compounding interest annually, quarterly, and continuously. However, they found that the Rule of 69 was the best approximation for monthly. They came to this conclusion by looking at percentage errors calculated for each Rule. They found that the Rule of 72 had larger percentage errors than the others. The grader asked why use it then. The student replied that it was and overestimation, whereas the other underestimated the doubling times. An interesting note--these students confused compounded daily with compounded continuously.

The question of sensitivity came up again. The students said that the function became more sensitive with successive lowering of the interest rates. They looked at small changes around the interest rate points. They then used the "slope formula" to get the "rate of change".

Group 6: 10:10 a.m.

One student started by reading Dr. Bilkins' speech and then explaining the problem. They computed the actual doubling times as well as each of the approximations for different compounding units. They found the percentage error for each approximation. Their findings was that Rule of 72 did best for annually and quarterly, Rule of 70 was best for semi-annually, and Rule of 69 was best for monthly and continuously. Rule of 72 was not the closest approximation for each, but it is the only on that overestimates for each compounding unit. That is why the Rule of 72 is used most often.

Another student talked about rate of change. When increase the interest rate, your rate of change increases, The greatest change happens form 2% to 8%. Overall it is best to invest at 20% because the time to double is much shorter. The percentage increase is smaller, but then so is the investment time. The grader asked how the student computed this rate of change. She said that she subtracted the time at 2% from the time at 8% and then divided by 6%. The grader asked if this was the average rate of change and she said yes.

Finally the grader asked how they chose which approximation was best. the said that they looked at the percentage errors and whether or not you overestimate or underestimate. What else did you take into consideration? One student answered that the approximation was way off at the endpoints but fairly close in the middle. Then another student said that was true for Rule of 72. However, for the others, they were closer at lower interest rates. As interest rates increase, the percentage error increases.

Group 2: 11:15 a.m.

This group begin by saying that the Rule of 72 was a rough estimate and that the exact doubling time used compound interest formulas. They produced a transparency that showed the exact doubling time compounded annually, semi-annually, quarterly, monthly, daily and continuously for interest rates between 2% and 25%. The next transparency showed the calculation of the approximations rules. The third transparency showed the calculated percentage error for the Rule of 72. The grader asked if they prepared percentage errors for the other rules. They didn't. Referring back to transparency three, they said that a "+" sign in front of the percentage error indicated an overestimate; whereas a "-" indicated an underestimate.

The next transparency had the following:

Interest Rate	Rate of Change
2%	-16
8%	-2
14%	-.4
20%	-.2

The grader asked if they had graphed their data. They said that the hadn't. How did they get the rates of change without looking at the graph of the data. The students said that the just worked with the numbers and imagined that they had the graph. The grader asked how they computed the rates of change. They said that they looked at small areas around each interest rate--like 2.0% to 2.01%. This didn't satisfy the grader. He asked again how they did this without a model. The students said that they didn't need a model. They used the formula

$$\text{Slope} = (2.01 - 2.0)/(34.61 - 34.53)$$

The grader asked if the graph was a line or a curve. One student answered that it was a line and then stopped to say that no, he was going to say that it was a curve. The grader again asked rather incredulously if the didn't plot points and compute a model. They again said that they didn't. Then one of the students said that they should have, but hind sight is 20/20. It seemed like a good thing to do. However, he said that these (their findings) were right though and that he would stand by their decisions. The grader said that he needed to be convinced and that he wasn't sure what they were doing.

Super Highway Project:

Group 1: 11:15 a.m.

This group was comprised of two males and one female. They began their presentation by showing a poster size map of Europe and tracing the route that the new Super Highway would take, pausing to show where the

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toll booths would be located. (The route that they traces followed the path of an existing highway that goes through the European countries designated by the project outline.)

They proceeded to discuss how they constructed their model for stopping distance for vehicles on this highway. Their data came from a commercial drivers manual (for 18 wheelers) that is put out by the Highway Patrol offices. Their rational was that the stopping distance for a heavy truck in rainy conditions would represent the maximum stopping distance for any vehicle. Stopping distance for such trucks in the manual was presented as a function of perception time, reaction time, and braking distance. To determine what speeds they should look at, one of the students called AAA to see what the average cruising speed was for the Autobahn. The found it to be around 95 mph. With this information, they chose to start their data set at 50 mph and conclude at 110 mph. This data set was done in standard measurement units and then translated into metric units. They plotted points and found that a quadratic function best modeled their data.

After looking at their model, they realized that some high speeds that could occur on this highway would lie outside of their data set. They contacted the SC Highway Patrol and asked if the extrapolations that they made from their model made sense. An officer there said they looked reasonable. Finally, the information from their model suggested that they should place the warning light .175 miles away from the toll booths. However, the students wanted to give themselves a buffer. They decided to add extra distance to this figure and place the lights .38 miles from the booths. This was done to take into consideration poor road conditions, worn tires and faulty brakes. Other safety precautions were: signs warning drivers to be prepared to stop before the flashing lights (in multiple languages), signs indicating toll amounts that would reinforce the idea that the driver must stop ahead, and grooves in the pavement to warn the drivers to slow down.

When discussing rates of change, the students used speeds for 75, 95 and 110 miles per hour as representatives of slow, medium and fast speeds. They constructed a functions of velocity over time. They said that the change was decreasing.

The grader was very interested in there choice of data source. They discussed this at length. The other question that the grader had was how did the students pick a quadratic model for their data. The students said that they looked at differences. The percentage differences tipped the scale. The grader was very impressed with this group and gave them perfect scores.

Group 3: 8 a.m.

This group began by giving a very brief introduction of the problem. They said that they contacted a professor in the Language department at Clemson who traveled in Europe occasionally. He said that traffic on the Autobahn goes as fast as a car can go. *Motor Trend* magazine approximated this to be around 222 km/hr. One student said that in the next five to ten years, people would have access to computer cars that could travel at speeds around 378 km/hr. This figure is what they kept in mind as their maximum speed. However, their data set ranged from 16 km/hr to 113 km/hr. They got this data from a 1993 edition of Automotive Encyclopedia. They decided to exclude the reaction time factor from their stopping distance data because warning lights would prepare people to stop.

They made a scatter plot of their data and tried to fit a model to it which could be used to extrapolate for higher speeds like the 378 km/hr figure above. They considered cubic, exponential and quadratic models. At first they thought the model should be exponential because it looked like the best fit according to the graph overlay. But after looking at the extrapolation values, they decided it would be ridiculous to use this model. They looked at the extrapolation values for both the quadratic and cubic models. Both sets of values looked reasonable. At this point they were unsure as to which was the best model. They decided to consult a physics professor at the university. He said that according to the laws of physics, the quadratic model would be best. He then showed them how this was true by using physics formulas that relate initial velocity to acceleration.

The student graphed their model on a poster, using both US customary and metric measurement units. From this model, they decided that the signs should be placed 1.93 km before the toll booths. This figure would be multiplied by 1.5 to take into consideration for wet road conditions. Other safety measures mentioned were a secondary warning system, run-off ramps 50 and 100 feet before the toll booths and bad weather signals.

Rate of change was then addressed. One student explained that for a speed of 378 km/hr, if you add an additional 1 km, it will increase the stopping distance by approximately 11. 46 km. They then read the press release and concluded by displaying a poster that showed the consequences (a wreck) if the warning lights were placed too late.

This presentation took longer than most. The grader did not have alot of time for questions. He did ask about the metric data and the US customary data. He wondered if they were the same data set or if they actually were taken from different sources.

Group 3: 11:15 a.m.

After a brief introduction of the problem, this group began showing charts of data that they collected. The grader interrupted them and asked for their data source. One student said that it came from the American Highway Practice which had a copyright of 1942. The grader had this incredulous look on his face. He asked if this data represented what happens today. One of the students answered, "Yes, sir." Then he asked if they thought that cars had changed any since 1942. Then he stopped and asked them to continue with the presentation.

The actual data ranged from 30 mph to 70 mph. They made a scatter plot of this data and then extrapolated to complete the data chart for values of 80, 90, 100, 110 and 120 miles per hour. Again the grader stopped the group to point out that their chart was misleading since the actual data was from 30-70 mph. Their reason for extrapolation was that someone in the Physics department told them that the average speed on the Autobahn was around 100 mph.

The group decided on a quadratic model due to what they called a "constant coefficient of friction" on dry level ground of .4. They considered their highest speed to be 120 mph. From their model, they concluded that the light should be placed 1200 ft from the toll booth. However, to give themselves buffer they decided to place the warning light 2000 ft from the booths.

The graders first question at the end of the presentation was about rate of change. One of the students gave the formula for y'. The explanation was that y' was "miles per mph" or "miles/mph". So at 70 mph the rate of change is 11.68 mile/mph and for 120 mph the rate of change is 20.04 miles/mph. The next question dealt with the model itself. The grader wanted to know how the group knew it was quadratic. They said it was the best fit from the differences. The first differences were constant. The grader asked if it were the first differences or the second differences that were constant. One student said it was actually the second differences. The grader asked if they looked at other models. They said that they looked at all of them. The grader asked what the differences told them. The student said that they were relatively constant. They were then asked if the had looked at percentage differences and the students replied that they had not.

The grade concluded by telling them that extrapolation in this case was very dangerous. The reason for this is because of their data was based on cars built in the 1930's. This caused their project to be fundamentally flawed.

APPENDIX III-L

Evaluation Test Items and Answer Key

Calculus Concepts: An Informal Approach

Volume I

Test Questions
and
Answers

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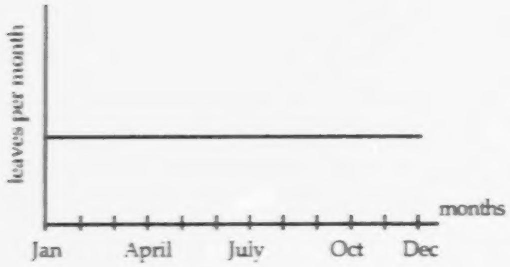
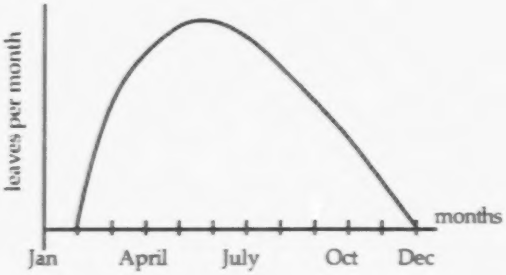
Informal Calculus I
Problem 1

Consider a tree on which new leaves grow in the spring, the number of leaves is at a maximum in the summer, and the leaves fall off of in the fall. Which of the following graphs best describes the rate of change (derivative) of the number of leaves on this tree over a one-year period?

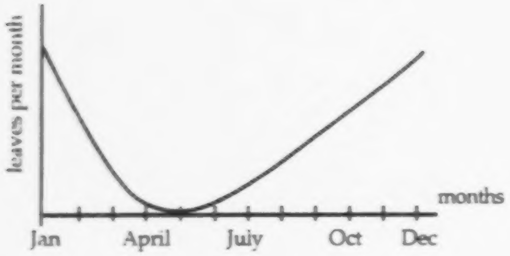
a.



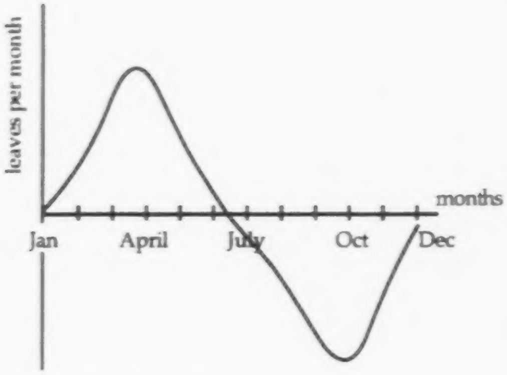
b.



c.

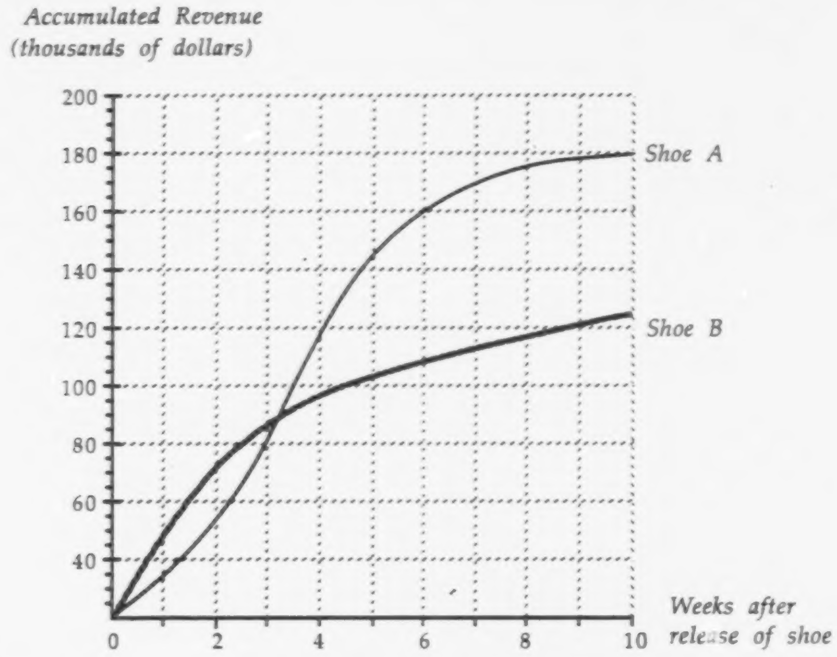


d.



Informal Calculus I
Problem 2

Two new running shoes were introduced by rival manufacturers at the Sports Shoe Show. The figure below shows the graphs of the accumulated revenue (in thousands of dollars) for each shoe as a function of the number of weeks since its release at the Sports Shoe Show.



a. Which shoe had brought in the most revenue by 2 weeks after release?

b. Which shoe's accumulated revenue was growing the fastest 5 weeks after release?

c. When was Shoe A's accumulated revenue increasing the fastest?

d. Over what time interval was the rate of change (derivative) of Shoe B's accumulated revenue positive?

e. Estimate how fast Shoe A's accumulated revenue was growing 6 weeks after its release. (Include units of measure with your answer.)

Informal Calculus I
Problem 3

The first day of this school year each child was sent home with information about free or reduced rate meals. This information included the income chart below and the statement "If your total household income is at or below the amounts on the income chart, your child may receive free or reduced meals (40¢ vs $1.15 daily for lunch and 20¢ vs 50¢ daily for breakfast)."

INCOME CHART

Household Size	1	2	3	4	5	6
Annual Income ($)	12,895	17,446	21,997	26,548	31,099	35,650

a. What is the rate of change (derivative) of annual income with respect to household size? (Give units of measure.) Briefly explain how you arrived at your answer.

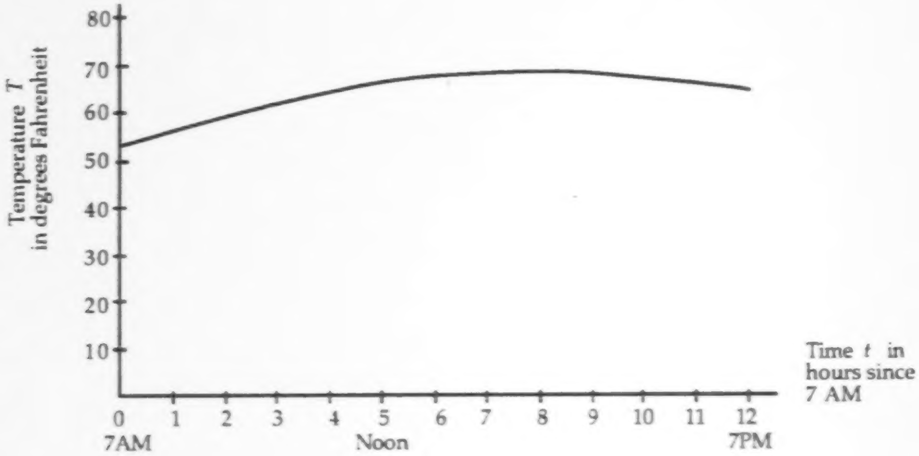
b. If the Johnson household's annual income is $44,000 and their household consists of 8 persons, extend the income chart to determine if the Johnson children are eligible for free or reduced meals. Explain how you arrived at your decision.

Informal Calculus I
Problem 4

The air temperature during a period of 12 hours is given by the model

$$T = 52 + 4t - 0.25t^2$$

where t is measured in hours after 7 AM and T in degrees Fahrenheit.



a. During what hour is the temperature increasing most rapidly?

b. At what time is the temperature not changing?

c. What is the average rate of change of temperature between 10 AM and 2 PM?

Informal Calculus I
Problem 5

The profit, in thousands of dollars, realized by a certain corporation from 1980 through 1993 can be modeled by the function

$$p(x) = {}^{-}1.48x^3 + 26.64x^2 - 48.84x + 33.81$$

thousand dollars where $x = 0$ at the beginning of 1980, $x = 1$ at the beginning of 1981, etc.

a. How fast was profit changing at the beginning of 1990?

b. In what year was profit growing most rapidly?

c. What was the maximum amount of profit realized?

Answers
Informal Calculus, Volume I

Problem 1

Concepts: Understanding rate of change, relation of the graph of a function to its slope (derivative) graph

Answer: *d*

Problem 2

Concepts: Graph reading, slopes of tangent lines, inflection point, relation of derivative graph to function graph, numerically estimating the slope of a tangent line

Answers:

a. Shoe B

b. Shoe A

c. 3 weeks (may vary slightly)

d. 0 to 10 weeks

e. 12,000 dollars per week (answers will vary slightly)

Problem 3

Concept: Constant rate of change

Answers:

a. 4551 dollars per person because the increase in annual income for each additional person in the household is the constant value 4551

b. For each additional person in the household, the annual income in the chart increases by \$4551. Thus, for 8 persons in the household, the maximum income for eligible house holds is 35,650 + 4551(2) = \$44,752. Yes, they are eligible.

Problem 4

Concepts: Slope of tangent line, derivative formulas, slope of secant line

Answers:

a. During the first hour (between 7AM and 8AM)

b. 3 PM

c. 1.5 degrees per hour

Problem 5

Concepts: Optima and inflection points, derivative formulas

Answers:

a. 39.96 thousand dollars per year (\$39, 960 dollars per year)

b. 1986

c. \$750.13 thousand dollars (\$750,130 dollars)

APPENDIX III-M

Evaluation Test Data Tables

	Calculus Concepts: An Informal Approach			
		Test Questions		
	Clemson	Spring 1995	N = 271	
	Correct	Correct, Wrong Units	No response	Incorrect
Problem 1	111			160
Problem 2a	262			9
Problem 2b	266			5
Problem 2c	220			51
Problem 2d	120		1	100
Problem 2e	142	15		114
Problem 3a	191	60		20
Problem 3b	240	6	1	24
Problem 4a	156			103
Problem 4b	167	1	12	103
Problem 4c	150	19		102
Problem 5a	121	25		125
Problem 5b	130	3	1	137
Problem 5c	130	11	1	129
	Correct	Percent Correct		
Problem 1	111	41		
Problem 2a	262	97		
Problem 2b	266	98		
Problem 2c	220	81		
Problem 2d	120	44		
Problem 2e	142	52		
Problem 3a	191	70		
Problem 3b	240	89		
Problem 4a	156	58		
Problem 4b	167	62		
Problem 4c	150	55		
Problem 5a	121	45		
Problem 5b	130	48		
Problem 5c	130	48		

Calculus Concepts: An Informal Approach

Test Questions

Kennesaw Spring 1995 N = 27

	Correct	Correct, Wrong Units	No response	Incorrect
Problem 1	5			22
Problem 2a	25			2
Problem 2b	26			1
Problem 2c	24			3
Problem 2d	16			11
Problem 2e	13	3		11
Problem 3a	23	1	2	1
Problem 3b	22		2	3
Problem 4a	0			0
Problem 4b	0			0
Problem 4c	0			0
Problem 5a	26			1
Problem 5b	11	11		5
Problem 5c	10		2	15

	Correct	Percent Correct
Problem 1	5	19
Problem 2a	25	93
Problem 2b	26	96
Problem 2c	24	89
Problem 2d	16	59
Problem 2e	13	48
Problem 3a	23	85
Problem 3b	22	81
Problem 4a	0	0
Problem 4b	0	0
Problem 4c	0	0
Problem 5a	26	96
Problem 5b	11	41
Problem 5c	10	37

Calculus Concepts: An Informal Approach

Test Questions

East Texas Spring 1995 N = 16

	Correct	Correct, Wrong Units	No response	Incorrect
Problem 1	5			11
Problem 2a	15			1
Problem 2b	16			0
Problem 2c	12			4
Problem 2d	8	2	1	5
Problem 2e	6	2		8
Problem 3a	7	7	1	1
Problem 3b	16			0
Problem 4a	2		1	13
Problem 4b	4	1	1	10
Problem 4c	2			14
Problem 5a	4	1	1	10
Problem 5b	3	2	1	20
Problem 5c	2	2	4	8

	Correct	Percent Correct
Problem 1	5	31
Problem 2a	15	94
Problem 2b	16	100
Problem 2c	12	75
Problem 2d	8	50
Problem 2e	6	38
Problem 3a	7	44
Problem 3b	16	100
Problem 4a	2	13
Problem 4b	4	25
Problem 4c	2	13
Problem 5a	4	25
Problem 5b	3	19
Problem 5c	2	13

Calculus Concepts: Traditional Approach

Test Questions

East Texas Spring 1995 N = 20

	Correct	Correct, Wrong Units	No response	Incorrect
Problem 1	2			18
Problem 2a	20			0
Problem 2b	20			0
Problem 2c	8			12
Problem 2d	1	5	3	11
Problem 2e	0		4	16
Problem 3a	6	7	6	1
Problem 3b	13		4	3
Problem 4a	4		3	13
Problem 4b	3	1	5	11
Problem 4c	1	1	2	16
Problem 5a	0	1	12	7
Problem 5b	0	0	14	6
Problem 5c	1	0	10	9

	Correct	Percent Correct
Problem 1	2	10.00
Problem 2a	20	100.00
Problem 2b	20	100.00
Problem 2c	8	40.00
Problem 2d	1	5.00
Problem 2e	0	0.00
Problem 3a	6	30.00
Problem 3b	13	65.00
Problem 4a	4	20.00
Problem 4b	3	15.00
Problem 4c	1	5.00
Problem 5a	0	0.00
Problem 5b	0	0.00
Problem 5c	1	5.00

Calculus Concepts: An Informal Approach

Test Questions

	Greenville Tech	Spring 1995	N = 33	
	Correct	Correct, Wrong Units	No response	Incorrect
Problem 1	17			16
Problem 2a	33			0
Problem 2b	33			0
Problem 2c	20			13
Problem 2d	23		1	10
Problem 2e	18	3		12
Problem 3a	29		1	4
Problem 3b	30			3
Problem 4a	24	1	1	8
Problem 4b	24		1	9
Problem 4c	20	7		6
Problem 5a	20	3	1	10
Problem 5b	15		1	18
Problem 5c	13	3	4	7

	Correct	Percent Correct
Problem 1	17	52
Problem 2a	33	100
Problem 2b	33	100
Problem 2c	20	61
Problem 2d	23	70
Problem 2e	18	55
Problem 3a	29	88
Problem 3b	30	91
Problem 4a	24	73
Problem 4b	24	73
Problem 4c	20	61
Problem 5a	20	61
Problem 5b	15	45
Problem 5c	13	39

Calculus Concepts: An Informal Approach

Test Questions

Elon Spring 1995 N = 25

	Correct	Correct, Wrong Units	No response	Incorrect
Problem 1	12		1	12
Problem 2a	25			0
Problem 2b	23	1		1
Problem 2c	20			5
Problem 2d	14		1	10
Problem 2e	13	2	1	9
Problem 3a	21	4		0
Problem 3b	23			2
Problem 4a	12			13
Problem 4b	18			7
Problem 4c	14	1	1	9
Problem 5a	10	1		14
Problem 5b	13			12
Problem 5c	15			10

	Correct	Percent Correct
Problem 1	12	48
Problem 2a	25	100
Problem 2b	23	92
Problem 2c	20	80
Problem 2d	14	56
Problem 2e	13	52
Problem 3a	21	84
Problem 3b	23	92
Problem 4a	12	48
Problem 4b	18	72
Problem 4c	14	56
Problem 5a	10	40
Problem 5b	13	52
Problem 5c	15	60

	Calculus Concepts: An Informal Approach			
		Test Questions		
	College of Charleston	Spring 1995	N = 119	
	Correct	Correct, Wrong Units	No response	Incorrect
Problem 1	46			73
Problem 2a	118			1
Problem 2b	117			2
Problem 2c	103			15
Problem 2d	84		2	33
Problem 2e	68	2		49
Problem 3a	83	14		22
Problem 3b	104		1	14
Problem 4a	63	1	1	54
Problem 4b	80	4		35
Problem 4c	60	3	2	54
Problem 5a	64	8	1	46
Problem 5b	64	1	5	49
Problem 5c	44	4	6	65

	Correct	Percent Correct
Problem 1	46	39
Problem 2a	118	99
Problem 2b	117	98
Problem 2c	103	87
Problem 2d	84	71
Problem 2e	68	57
Problem 3a	83	70
Problem 3b	104	87
Problem 4a	63	53
Problem 4b	80	67
Problem 4c	60	50
Problem 5a	64	54
Problem 5b	64	54
Problem 5c	44	37

Calculus Concepts: An Informal Approach

Test Questions

Lenoir-Rhyne Spring 1995 N = 27

	Correct	Correct, Wrong Units	No respons	Incorrect
Problem 1	15			12
Problem 2a	26			1
Problem 2b	27			0
Problem 2c	26			1
Problem 2d	21	1	1	4
Problem 2e	5	4	1	17
Problem 3a	16	10	1	0
Problem 3b	26		1	0
Problem 4a	13	3		11
Problem 4b	14	5		8
Problem 4c	6	5		16
Problem 5a	10	2		15
Problem 5b	8	2	3	14
Problem 5c	6	4	3	14

	Correct	Percent Correct
Problem 1	15	56
Problem 2a	26	96
Problem 2b	27	100
Problem 2c	26	96
Problem 2d	21	78
Problem 2e	5	19
Problem 3a	16	59
Problem 3b	26	96
Problem 4a	13	48
Problem 4b	14	52
Problem 4c	6	22
Problem 5a	10	37
Problem 5b	8	30
Problem 5c	6	22

APPENDIX III-N

Student Comments

Critique of Math Science 102 and 207

These two math classes took a very different look at calculus in a way that related very well with my learning style. I learn best by understanding how a certain thing is used in real life. That is exactly the whole purpose behind the 102 and 207 procedure. The information and formulas are given to the student in a real life manner and we have to interpret the data and crunch the numbers in order to make the data understandable to someone we might be presenting it to or even to ourselves.

This is where the presentations in 102 came in. This really made me realize that this course was truly preparing me for the future in business. It all made sense in my head that I was doing these formulas and calculations in order to clarify and make the sales pitch to an investor, so in other words you had to understand what you were saying.

To sum up everything I feel that the way 102 and 207 are set up is great. It helps me to understand what I am doing and many time I pick up the procedure on the first try because it is practically common sense. I think it should the program should stay at Clemson (even though I didn't do all that great), and I am sure it will quickly become popular at other schools. Thanks to all the professors for all there hard work in locating all the recent data and aplying it to the calculus problems.

Over all, I felt that these classes were effective in that it established a certain way of thinking. Realistically, I will probably never use most of what we learned in the class, but I will use the thought processes that were needed to complete this class. As far as the calculator based issue, I have mixed feelings about it. On one hand I thought it was good and made the class much easier, though on the other hand it took me a while to really learn to use the calculator which hurt me in the beginning.

I have never taken the more traditional courses where the calculator was not used, or derivatives taught with rules and regulations to follow. Although this method was used for so long, using the calculator is allowing students to learn and practice in a more computerized society. All math sections and applications will eventually become computerized, so I think that learning it slowly now will really end up helping everybody in the long run. Students are taught to understand and interpret the math applications and not just how to perform the tasks at hand and get an answer.

I feel that a good job has been done in the revamping of the courses' curriculum. The material is presented in a logical and coherent fashion, and makes it easier for the nonscience major to be successful. The use of the TI-82 dramatically reduces the mundane and unnecessary memorization that the course normally requires. With a few exceptions, MTHSC 102/207 is nearly flawless for the committed student.

In taking my first two math courses in business calculus this year I feel I have learned much more about calculus. I had very little experiences with calculus in high school, so in 102 I had trouble at first catching on with derivatives. I remember doing those type problems and wondering how I would ever use these skills later on in my working career with a textiles major. Later on in the class I had a had time with the final part of our project with the Carolina-Clemson T-shirts. I didn't understand how to do the second derivative and apply it with the cost, demand, revenue, and profit functions to find the maximum profit. As far as the project, I felt there could have been a better example to use or maybe everyone could have been given the same information on how many students there were and all of the other things we were supposed to get on our own. Also I felt my instructor did not go over it enough in class.

This semester in 207 I have done much better, and I have enjoyed taking the class. I was glad to find out that we did not have a project to do this semester. As the class progressed through the semester, I began to see how all of this mess comes together into something that I will be able to use in the future of my career. I felt my teacher has done a good job in explaining the material.

As far as the use of the calculator in these two classes, I feel that it has been a great help. I think that it eliminated a lot of long handwritten problem solving. I am glad that I was able to use the TI-82 for these classes and without it I would have been in some trouble.

Critique of Calculus Reform Class

I took MTHSC 102 elsewhere as a traditional math class. It was very challenging for me. It took most of the semester before I figured out what we were doing and why. I like the Calculus Reform Class much better. I have a good grade, I understand what's going on and when the things we learn will actually be used in "real life". Most students, at one time or another, ask "when will I really use this stuff?". I took the approach for MTHSC 102 that I really would never use it, I had to take the class because it makes me think and it's required. In the reform class, I see that what we learn can be applied to the real world. Although, there are usually easier, more expedient ways to get the desired results (like a financial calculator or a computer program) that will probably be utilized in a work environment.

All in all I think the course was very effective in what it was trying to do. The goal of the course was to give business majors an idea of how to apply calculus concepts to business. At first I thought there would be no way to apply math to the "real world" but I can honestly say that this course has shown me that you can apply math to business. As a business major I would much rather take this course than waste my time with a high level math course that will essentially do absolutely nothing for me other than stress me out. This course was very fair along with teaching me a lot.

The project I did last semester in 102 was a good way of applying what I learned in the classroom to a real life scenario. I feel that the presentation that accompanied this project forced each member of my group to explain what they had learned in the classroom.

Calculus Evaluation

Calculus is a difficult math class if it is not one of you strong subjects. Since I took it in high school as a senior with out a calculator it was much easier to understand in college. There is less frustration of dealing with the algebraic equations because you enter them into the calculator. I have done much better in my math classes in college than I did in high school for the simple fact that you make less mistakes and the course is set so that the calculator will be to your advantage. There was less figuring out of equations and more focus on functions and graphs. To me that makes more sense because you will use that in everyday life. I think the book was very easy to understand and that helps when math is not your favorite subject. The simple equations as examples helped to put a foreign idea into perspective. Math 102 and 207 as a course were set up in a way that students could understand and do well in. That is an important factor because we have to many courses were the class is almost impossible to understand and the book does not help and neither does the instructor.

To: Don LaTorre
From: Heidi Staebler, East Texas State University

Student Comments on the Survey

- I was scared of calculus - that word. But this course was much easier than any math I've ever had. It's more logic than x's and #'s.

- good Course, fun to learn

- I feel that I needed a few more examples in some of the sections. I got a little lost toward the end of the course.

- This course is a good learning experience for students with any kind of business major. It prepares them better for real life situations and is beneficial to train the thinking processes to deal with situations.

This person marked "easier" for 7.) but commented - "but still hard in places."
He/she also wrote "don't know" for 8.) but commented - "I think I learned more in this class than I would have in a traditional section of this course."
Finally this person wrote the following: - More descriptive examples in the book would be helpful to understand derivatives. I found the readings in these chapters very confusing (I could not tell how to do the problems just by reading.) I had never been exposed to the idea of derivative, therefore, I think some step by step examples with numbers instead of words, would have been more helpful in the chapter about product rule, chain rule, etc. Overall, I've done better in understanding this material than I usually do in a math course.

- This is a god send course for those who are not algebraically inclined and who get very intimidated by rules and procedures.

- The graphics calculator should be used in the classes below and above this class to learn how to model more efficiently and more complex problems also should tie in with statistics courses.

From Heidi Staebler East Texas State Univ
To: Don La Torre

More Student Comments on the Survey

In the following comments I have added some comments of my own. My comments are in {}. Heidi

- I do believe that the graphing calculator is an invaluable tool. However I have conflicting feelings about the use of a calculator in general. I feel that a student needs to know how to use a calculator, but often the use of calculators have a tendency to make us lazy, then when we forget to bring it, or the battery goes down, there aren't many of us who can do things by hand. On question #8 I put learned less because of the following reason's. I have had Calc. I traditional { I think this individual means mainstream not business calculus}. I didn't learn any more but I didn't learn any less and I do have an opinion so I put learned less. Something I would like to see having learned the product rule in the traditional manner. I have memorized the product rule you have in the book, but I will have to go back and relearn it in the traditional manner to go on with my Calc. II and Calc. III. I have a business major with math minor. When I tried to remember the quotient rule I got it backwards, and one of the math professors said it was because of the way I memorized the product rule. He said it was backwards, but when I showed him my book he agreed that I had memorized it in the proper manner.

{I think this student is complaining because of learning $(fg)' = f'g + fg'$ in this course rather than something which is somehow more similar to the numerator in the quotient rule?! If this is a source of difficulty for the student in Calc. II and III, then the student is probably going to have some very major difficulties anyway!}

[✓ I learned more]   no opinion   I learned less

Thank you for your time! We will also appreciate any thoughtful comments you may wish to mak

Comments: You cannot build a house without a good foundation. I did well in my other math courses here at LR. This course jumps into Calculus to fast. The models and data are wonderful, but not the heart of calculus. We must have a strong background on interpreting graphs and data to be effective in this course. Spend more time on the fundamentals

The book really needs some work! Must have answers to questions for student reference. Examples of problems similar to the activitie should be given. This book and course has been detrimental to my GPA (3.87) even though I did put in a lot of effort.

Comments: I have always been an A student in math. This book did not have enough examples to refer back to. Need modifications on I learned less

any thoughtful comments you may wish to make.

equire more algebra than
ell in the course considerin
ince I had algebra & had
easy rules.

he book needs a
nswer. This is so the
her work over night.
asic information and
the students intelligence.

ook is good.
Good Job.

with examples and the examples
nderstand.
learning the principle of this

: Survey Sheets.

back of the book in order to check those unsure about while doing
too hard to follow along the examples given were too difficult to
w material. New material examples should be basic.

tuations. These made the material less abstract and more
se I had very little exposure to Calculus. I do feel like this is a

engineer fields.

s to do the same problem, this makes it very confusing.
explain the material well, and that the instructor needs to clarify

fore. the other 2 times were the traditional way and I simply
at all. With the new course I understand it perfectly and I can
the first time. The first 2 times I wondered when I would ever

w if homework is done correct or incorrect by student.

iningful if a study guide and answer are provided to check

ng with political issues is not relevant to the study of math.
at affected the data?" The text should also include more example
proach to calculus, but the text is lacking greatly. The text needs
examples given and the homework problems. Don't always
ble to figure out these unfamiliar concepts on his/her own.
o teach themselves. We pay instructors to instruct and not to sit
t the students to learn the concepts on their own but also to teach

e text made understanding the material much easier. I applaud
e text. When I transfer to Clemson University I hope that my
se a text much like the one I used in Math 130 at Greenville

END END OF ROLL

It would be helpful to have the answers to the problems in the text so we could check our answers.

It would be nice if the answers to some of the problems were in the back of the book.

A good way to improve the book would be to supply answers in the back of the book whenever possible. With answers students can check their work and have a goo feel for how they are doing.

6-7

5. (Answer this question ONLY if your class studied integrals.) Our approach to integrals was to focus on understanding and interpreting the integral as *the result of change*, instead of as a collection of rules and procedures for evaluating integrals. To what extent do you agree with this approach?

Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree

6. We believe that a *graphics calculator* can be a valuable tool in helping students to learn and to do mathematics. To what extent do you agree with this?

✓				
Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
~~||||~~ |		|		

7. What is your assessment of the *level of difficulty* of this course, compared to your perceptions of the more traditional version of this course?

	✓			
much easier	easier	no opinion	harder	much harder
	|	||||	||	

8. Are you *learning more, or learning less*, than comparable students in more traditional sections of this course?

✓		
I learned more	no opinion	I learned less
~~||||~~ |	|	

Thank you for your time! We will also appreciate any thoughtful comments you may wish to make.

Comments: I took the more traditional version of this course last semester. It was a nightmare, and I have done alot better in this course.

APPENDIX III-O

1994-95 Pilot Site Interview Summaries

Interviews conducted July 28-30, 1995
Calculus Concepts Conference, Clemson SC

University of South Carolina - Aiken

USC-Aiken has about 3500 full time students in their undergraduate program. The mathematics department, which has approximately 13 faculty, also houses computer sciences, astronomy, physics, and a transfer engineering program. Up until this point, the university has offered only one 3-hour credit course in Business Calculus. They offer two to three sections in the fall, three sections in the spring, and one class in the summer.

They ran a pilot section in the Spring and Summer of 1994. No projects were used during these classes in order to get into integration topics at the end of the course. Students' grades were better in the pilot classes using the Clemson Project than they were in the traditional course. The instructor had previously taught Business Calculus with the computer program MATHEMATICA and found that was not the way to go. He had a good experience with the graphing calculators.

USC-Aiken will be teaching two project sessions this fall. The instructor that ran the first pilot course will teach one section, and the other will be taught by another instructor that attended the conference. There are hopes to incorporate student projects this semester.

Some of the faculty at USC-Aiken have had some opposition to using these materials. One of these professor's area of interest is engineering. He is currently finishing up his Ph.D. in this area. He taught a traditional Business Calculus course for the first time last Spring. He said his opposition to the Clemson Project came from the fact that this was not the way he learned calculus. However, he is glad that he came to the conference. He said that although he is not teaching the course this fall, he has had to rethink the way he will be teaching Trigonometry this fall.

Elon College

Elon College is a small liberal arts school of about 3500 students. It has an MBA program and recruits mostly form the Northeast. The Math department invests in trends: groups, writing, technology. They require calculators in all their classes now: HPs and TI-82s. Several people also use a statistics package called MINITAB.

The instructor that represented Elon went to Columbia for a workshop where Don LaTorre was giving a seminar. She asked if she could pilot it at her school. Elon has a four semester hour course in Business Calculus. Traditionally, most students in this course are business majors. Her course does not get into integration.

In the fall of 1994, this instructor taught the course. She did a project in which a professor from the Economics department assessed the students. They were very creative projects, but took up a lot of time. In the Winter term, another professor at Elon taught a short session course with the materials. He had a decent time with it. Instead of a huge project, he sent students to the library to find a data set of their own, model it, and make predictions. That Spring (1995), the head of the mathematics department, as well as Leon's conference representative, taught courses using the project materials. The department head did not like it, but gave no explanations why.

The representative said that she gets more comfortable with the materials each time she uses them. It takes a lot of work, but it is a profitable experience. To help her school get more understanding about the project, this instructor brought in Don LaTorre to do an interdisciplinary workshop at Elon. Those attending were mainly from the Math and Business Schools. He basically covered using the TI-82s in class, and gave an introduction to the project. He then did a presentation to the MBA people later that afternoon. The business school was quite receptive.

College of Charleston

The College of Charleston has about 8000 undergraduates and 100 graduate students. They have a one semester course called "Business Calculus" that is predominantly composed of business majors. There are six sections offered in the spring with about 40 students in each section. We are not sure how many sections are offered in the fall. They heard about this project through Iris Fetta. She did an invited workshop on using graphing calculators in mathematics classes at College of Charleston. She pulled examples from this project. The reason they looked into the program was that they thought Business Calculus needed to change. They really liked real data sets.

Three instructors piloted the course in Spring of 1995. These classes met three times a week for 50 minutes. One instructor said that they just got the materials and dove in. He covered through Chapter 5 and did a little integration. The preparation time was very demanding. He had no idea what to expect. He plans to teach the course again in the Fall. He will be the only one using the project materials at this time.

This instructor said that prior to the implementation of the pilot course, the Business Calculus course was very traditional. His overall impression of the pilot was that it was hard. It was not a smashing success. He had several students drop the course or transfer to regular sections. He had a hard time teaching with the calculator. One has to be careful not to substitute mindless calculator procedures for mindless algebra manipulation. To keep this from happening, he had students put labels on graphs and used test questions that made students think. An example is finding the maximum of a quadratic function. If one gets the wrong window on the calculator, the graph can look linear. He had some students finding the endpoint of that window as the maximum of the function.

A success he had occurred when one student's mother called the president of the college. She complained about the his/her failure in the 105 course. The instructors did a comparison. There was close to zero correlation between previous grades and grade in the project class. However, overall GPA had a better correlation.

The class is different. He did not want to use projects his first time teaching the materials to make sure he got through integration. He stated that he will do some this Fall. Traditionally, everyone at the College of Charleston has a uniform final exam. This time they sat down to try to make some questions for the final exam to be the same for all students. This was hard to do. He has applied problems whereas the others had more skill oriented problems. The ten Clemson question were not used by the other teachers because they felt they were not relevant to their classes.

This instructor does have some reservations about the project. He feels there needs to be more algebra. He believes in the power of abstract symbolism. He questions if they are selling some students short with this course. It has been the opinion of the department that it is our job to teach mathematics and the business department's job to teach the applications. Does the abstract imply application?

He feels that this course is harder than the traditional because of the word problems. However some mathematicians would look at the word problems and say that the materials were "watered down". Some students need this (approach) and some do not. Regardless of his reservations, he never wants to teach the traditional business calculus again.

Another instructor that piloted the materials got her masters in mathematics from Clemson a few years ago. She knew several people working with the project. That was one reason she went to both workshops held by the project staff at the ICTCM conference.

This instructor felt that one of the biggest hesitations they had with implementation was the fear of not having TI-82s. They had used a statistics package in the computer labs many times, but scheduling lab times was a burden. She felt she had the least amount of problems with the calculator of the three instructors. She definitely feels that giving a one hour credit course for learning the calculator is ridiculous. One learns as they go along.

She thought working in groups was a good idea. She said that her students were forced to work in groups because they had no solution key to homework problems and she didn't have time to go over them all the time. The math labs couldn't help these students with their problems, so they developed a good network between them as the semester passed. She did comment at this time on the large drop rate. Yes, many students dropped the class, but it was no worse than the traditional drop rate.

When working with materials she stressed units in the problems. They did not do any projects because it was too time consuming. She hopes that someone will do a workshop on how to do projects. Assessment for her came predominantly from tests and quizzes. You can only get two "thinking" problems on a 50 minute test. She had more success with daily quizzes. She highly recommends feedback to students and giving them a chance to rewrite problems.

We talked a little about communication for students and how important that was. She said her most valuable instruction tool in that area was modeling complete sentences on the board. It took a lot of time and gave you less time to cover more examples. That means that the students have to read more of the book. She felt that the initial text wasn't that easy to read for the weak mathematics student. She wants to look at the current version to see if it has gotten any better.

One thing she would do differently is stress the input/output diagrams from the beginning. This would help her students tremendously when they got to product rule and composition problems. She said she had a terrible time with her students getting them to recognize which was which.

She is really excited about teaching with the project materials next spring.

Greenville Technical College

Across the board implementation happened at Greenville Tech. Their first semester course has to cover integration rather thoroughly for transfer reasons to Furman and Clemson. They usually offer two or three sections per semester. The second semester did not use the project materials. There just weren't enough materials at the time. They supplemented from Goldstein, a traditional text. A lot of their students come from Furman. They are older masters students in business who had no calculus.

One of the instructors who was key in bringing the project materials to Greenville Tech said that it is a real improvement over the traditional. They are handicapped at Tech by transfer regulations of other schools, so they must modify the first section. They didn't do projects because of time constraints. He remembered thinking that this material was too trivial at first. Now he doesn't think that.

What he liked best was that problems were concrete enough. If students misrepresented the problem, there was no argument that they did so. He thought the material was really laid out well. He didn't give answers to the students. They didn't like this too much. They also weren't too thrilled with the fact that a question could have more than one answer.

He says that implementing the Clemson Project takes a good bit of work. He took about five to six times longer to prepare for this class than his others.

Lenoir Rhyne College

Lenoir Rhyne has approximately 1200 students. They have undergone a reformation in mainstream calculus over the past couple of years. First they used traditional book and wrote labs. This wasn't satisfactory. Then, they used the Harvard model for two years. Finally, this fall they will take a step back from Harvard and use Smith's book.

Business students have usually taken Linear Programming in the fall and then Business Calculus in the spring. (They may soon make both semesters Business Calculus.) For years the students hated Business Calculus. Classes ran around 20 students -- none over 30.

Lenoir Rhyne heard about this project at a regional MAA meeting in Tennessee. They only used the first semester material of the project for their pilot in the Spring of 1995. Two sections of the project was run by the instructor that we interviewed and another instructor not in attendance. The other instructor was too traditional with the book. The instructor I interviewed did not require calculators for this course. He used a statistics package in the computer lab for the modeling sections. The students did well.

He had some problems with the materials. He really would have liked to see a chart in the modeling sections that showed the given y and the predicted y. He felt that students would have benefited from this. He also would have liked to see the derivative of 2^x developed by use of difference equations since one can use the calculator to show what happens as h gets smaller. He wasn't impressed with the logistic model. The materials need to discuss where the model comes from and include some discussion of differential equations. He would love to have a quotient rule. One thing he did like -- he got an education about 'plow sulkies' from one of the applied problems in the text.

He said that evaluation was tougher. He gave tests in the computer room. He also gave a traditional test on derivative rules.

APPENDIX III-P

1994-95 Pilot Instructor Comments

To: Don LaTorre June 7, 1995
From: Richard Sauvageau
Re: Evaluation of Calculus Concepts, second semester

Dear Don,

Since I had the same students first semester as second semester, my second semester evaluation consisted of the students writing a narrative telling me of their feelings and judgment of the course. As can be expected, many said the same thing. I am summarizing many of their comments. (No attempt has been made to "clean up" the grammar or spelling)

" I absolutely loved this math class. I have to say that this was my favorite math class here at Staples. I learned a lot and enjoyed coming to class every day. The calculus I learned will help me anytime in the future."

" This was the most interestingly taught math class I have ever taken. Yes, it's hard and yes it gets increasingly tougher throughout the year, but in a weird way I didn't mind it. The book itself was good. Regardless of the grade, I really learned a lot as evident to my recent grades on the SAT II and college placement scores."

"I liked the small class size and the atmosphere provided by knowing everybody in the class well."

" This was the best math courses I've ever taken. The curriculum was original and unusual. I would have liked more real life applications like the ones in the first semester. I loved the calculator and this will help me next year in college."

" It was the most interesting math class I've ever taken. "

" This course took a new approach to math, it was helpful."

" I liked this course and I think you should definitely continue it next year."

My opinion:

At one point in the class, I passed out a traditional Calculus book to introduce the students some of the other topics they are likely to encounter in college. They unanimously hated it! Without exception, they asked where were the "real " life applications to some of these topics. One example was logarithmic differention. " This stuff is really stupid, when will we ever use it? Why don't they give us uses" was typical of the classes reaction.

Having taught AP Calculus for a number of years, I must say that the Calculus Concepts material is unique and interesting. I really enjoyed teaching and learning the material. The one word that I feel indicates the entire course is relevance. The students could see real applications to math. This is an area sorely missing in the high school curriculum. I don't feel the need or desire to go back to the traditional Calculus curriculum. I would be bored proving formulas and then assigning ten "plug it in and grind it out variety" problems for homework.

I feel the second semester needs some work. I missed not having projects to assign to the class. The projects were highlights of the first semester. The students presented their projects to assorted teachers and administrators. Nothing but rave reviews from all evaluators! I'm assuming the writers have worked on the second semester course and next year there will be improvements.

Over the course of the year, I'm shared the course with various high school math teachers and most of them indicated the need for a non-traditional AP curriculum for some of their students. All were very much impressed with the contents and the scope of the problems incorporated in the text. Several asked me for copies but I had agreed not to share the text with other schools. I know one school will be ordering the text for next year.

In summary, Calculus Concepts fills a much needed gap for some high school students. The students involved with this years pilot loved the course and enrollment for next year has almost doubled. I assure you, if the course weren't successful, enrollment for next year would be down. I look forward to the revised text and look forward to teaching it next year.

If I can answer any questions or you desire further information, I will be happy to supply it.

Sincerely,

Richard Sauvageau

Richard Sauvageau
Math teacher
Staples High School
Westport, CT. 06880

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June 7, 1995

To: Don LaTorre

From: Kathy Peters
Manchester Community Technical College

Re: Calculus Concepts

Teaching 2 sections of Applied Calculus from this book was a terrifically positive experience for me. Following this experience, I would not want to return to a traditional approach. In this one semester course, the students seemed to have a strong understanding of the concept of rate of change and were on their way to understanding the concepts behind the area under a curve when the semester ended. I have taught Applied Calculus many times on several campuses and have never seen the conceptual understanding that the 2 classes demonstrated this year. At the end of the semester, I asked them to do Problems 1,2,3,4,5 from Volume I and 1,2 from Volume II as an overnight assignment that was graded as one test. I was pleased with the results. On all questions, I asked for support and explanation of their answers.

Vol. I (1) Most chose correctly, but some explanations were incomplete.
(2) This question was easy for all of the students and most answered all parts correctly.
(3) The students uniformly answered part (b) correctly. In part (a), some said the units were dollars per household (instead of dollars per person).
(4) (a) Many gave one exact time, not an hour of time.
(b) Almost all answered correctly.
(c) Almost all answered correctly.
(5) This was the hardest question. Part (a) was handled uniformly well, but the support for the answers in (b) and (c) was awkward and incomplete on many papers, although most gave correct answers.

Vol. II (1) All knew how to extimate and most got the correct estimate.
(2) All knew the meaning of the 160, but some said the coin's value in 1991 was 660.

The student surveys indicated that the majority felt algebraic manipulation was de-emphasized, a modeling approach was used, and that the focus was on interpretation. All agreed that the use of the calculator was valuable. One-half of the 40 who responded felt that this course was easier than a

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To Don LaTorre	From T. Peters
Co. MCTC	Co. CS+E UConn
Dept. Math	Phone # 203-486-5045
Fax # 203-656-5230	Fax # 203-486-4817

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traditional course and that they learned more than students in a traditional course. Of the other half, most had "no opinion."

In my opinion, the course as I taught it from this book was very successful and I thank you for the opportunity. As I said earlier in this letter, I feel that to now teach from a traditional text would mean for me a giant step backward.

If you require more specific data, please let me know. I'm at home for the summer at 203-450-1629. Also, is there going to be a field test conference in July? If so, I think I'd like to attend - if I'm invited!

Thanks again!

Kathy Peters

May 18, 1995

Donald LaTorre
Department of Mathematics
Clemson University
104 Martin Hall
Clemson, SC 29634

Dear Don,

Thank you for allowing us to test the materials you have designed for teaching calculus to the liberal arts student. *CALCULUS CONCEPTS: An Informal Approach* allowed us to experiment with technology and focus on the conceptual aspects of this course. We chose to require a TI-82 for the course and used it during lectures. Some of our students had other calculators, but the majority used the "82." After overcoming the initial shock of having to learn how to use the calculator, many of my students came to love it. However, there were many who did not master the course or the technology. Our Business Majors are required to take this course, but they often do not have an adequate background in algebra, even for this course which is not algebra intensive. Despite your efforts to produce a text which could be used by the students, I think many where frustrated because there was too broad an approach to the topics and not enough examples. I enjoyed teaching using this approach, and I do think this is much better in teaching how calculus is more than an abstract term. What follows is my random thoughts for improving the text. We plan to teach a few sections in the fall, provided the text is available.

Although the book does not require a specific calculator, it would greatly improve the course if specific directions for the TI-82 were provided for the students. Designing handouts on calculator usage was a time consuming task which need revising after the first use.

In the initial portion on functions, modeling from word problems needed more emphasis. The sections on derivatives of products and composition functions could have gone smoother if they understood how to determine the model. Many students did not know how to form the model, even if the functions were given.

Learning to read graphs was a good point, but learning how to solve problems graphically by setting an equation to 0 and finding the x-intercepts could have been explained as well as finding points of intersection.

The modeling could have been more connected with the calculus, but I believe my students finally understood using a model as a predictor and how graphs suggest behavior more than charts and raw data. Determining the differences between linear, quadratic, cubic and exponential models gave them a significant understanding of graphs, (which surprisingly had not been acquired in other math courses). The logistic model should be introduced at the end of the course when the concepts of calculus have begun to solidify. The mechanics are too difficult initially.

I feel the hands on approach to graphing tangent lines was excellent!

The questions about derivative which were redundant really impressed upon the students the many interpretations of a derivative.

The problem with the "formulas" for derivatives came from the lack of practice. I would teach algebraic skills as an "aside" throughout the course so that the skills would be sharpened when they are needed. Recognizing composition functions and rules for exponents are tasks which need mechanical drill despite of my own dislike for drill.

On the final exam you asked a question which was not covered in the material but could have been inferred. You asked when the rate of change was the greatest and never discussed checking end points of intervals.

Teaching to visually recognize points of inflection before technically defining them was a great idea.

I liked the sections on integrals but we had to short cut them because we teach only a one semester course.

I found that some of my students were faking their understanding if they were required to write their answers in complete sentences and in context to the problem.

Look forwarding to seeing you this summer. Thanks again for all your help.

Sincerely,

Nancy Mauldin

Nancy Mauldin

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ELON
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THE MARTHA AND SPENCER LOVE SCHOOL OF BUSINESS

June 7, 1995

Mr. Brian LeKander
FIPSE
U. S. Department of Education
7TH & D Street S.W., Room 3100
Washington, DC 20202-5175

Dear Mr. LeKander:

I am writing in support of Professor Donald R. LaTorre's proposal to FIPSE for federal funding to disseminate exciting new materials in mathematics recently developed at Clemson University under a current FIPSE grant.

It was my pleasure to co-sponsor, with our Mathematics Department, a visit by Professor LaTorre to the Elon College campus last October. We had heard of the innovative nature of his work from one of our math faculty who was piloting his materials at Elon, and we were anxious to see it for ourselves.

Dr. LaTorre made two formal presentations at Elon, in addition to private conversations with many of our math, business, accounting and economics faculty. His first presentation focused on mathematical modeling using real-time data in calculus for business students. His presentation, using the marketing example, was so appropriate we asked him to modify his planned presentation for our 100+ MBA students to include the same exercise. It was enthusiastically received by both students and faculty. There has been, and continues to be, considerable interest and discussion among our faculty relative to the use of hand-held technology and modeling in our business school undergraduate courses.

I believe the materials developed at Clemson under Professor LaTorre's leadership are very valuable to students taking business subjects. The Clemson approach to calculus removes much of the traditional "fear-of-math" syndrome so prevalent in our society; therefore, FIPSE is to be commended for providing the developmental funding, and Professor LaTorre is to be applauded for bringing an outstanding "product" to our students.

In sum, I support, without reservation, Professor LaTorre's efforts in seeking additional funding to continue his excellent work.

Sincerely,

Richard H. Behrman

Richard H. Behrman
Dean

ELON COLLEGE, NORTH CAROLINA 27244 (919) 584-2366

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APPENDIX IV

Information for FIPSE

FIPSE's genuine commitment to quality in undergraduate education has made possible tremendous change at Clemson and helped us become national leaders in effectively using technology in undergraduate mathematics courses. The fact that most of the project participants have for many years dedicated their careers to innovative teaching methods and educational service significantly added to the success of this and other projects. We were mildly surprised when our external evaluation team was amazed at how closely the project materials follow the NCTM and MAA Standards because we did not design the project with those frameworks in mind. Long years of dedicated effort in teaching had evidently caused this to happen automatically.

The willingness to include less experienced team members on a large scale has also proved to be very rewarding to all of us. We have truly had a team effort on this project with each member of the author team contributing equally. This proved to be extremely important to the project when Don LaTorre, our director, was taken ill and not able to work with us this year. Even though competing publishers are trying to indicate that the "project is dead" without Don, nothing could be further from the truth. We have tremendously missed his leadership and dedicated efforts, but the author team has continued the project with great success. We know that it was our involvement at every stage along the way that made this possible. We strongly suggest that a team of truly involved participants rather than only one or two principals is very important to the success of any large scale project.

Even though our materials require the use of technology, we purposely did not refer to a specific calculator or computer in the text because the technology is changing so rapidly that the materials would soon be outdated. Yet, students require support in this area, especially when they are using personal calculators with which their instructor is not familiar. *Calculus Concepts* is unique among textbooks at this level in providing a technology supplement that follows the text discussion and gives instruction for each section of the text rather than being another version of the manual that comes with the calculator. This text supplement is much easier updated for popular new technologies than is the text. We strongly suggest to FIPSE that any future projects relying heavily on rapidly changing technology follow this pattern.

As previously mentioned in this report, projects must involve communication between the project participants and their intended audiences. It is not always an easy task to simply read about a new idea and successfully implement it as desired without support. The current work on our FIPSE dissemination grant is most certainly sustaining this theory.